Boundary integral method for bioluminescence tomography

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Abstract. Bioluminescence tomography (BLT) allows in vivo localization and quantification of bioluminescent sources inside a small animal to reveal various molecular and cellular activities. We develop a reconstruction method to identify such a bioluminescent source distribution using the boundary integral method. Based on the diffusion model of the photon propagation in the biological tissue, this method incorporates a priori knowledge to define the permissible source region, and establish a direct linear relationship between measured body surface data and an unknown bioluminescent source distribution to enhance numerical stability and efficiency. The feasibility of the proposed BLT algorithm is demonstrated in heterogeneous mouse chest phantom studies.

Bioluminescent imaging has the capability to reveal molecular and cellular activities directly, and can be applied to all disease processes in most small-animal models. It is very sensitive, and helps diagnose diseases, monitor therapies, and facilitate drug development.\(^1,2\) Bioluminescent imaging has been in a planar mode and largely a qualitative imaging tool.\(^3\) With bioluminescence tomography (BLT), 3-D localization and quantification is enabled of a bioluminescent source distribution inside a living small animal such as a mouse.\(^4,5\)

In bioluminescence imaging, photon scattering predominates over absorption in the biological tissue in this spectral range of interest. As a result, a significant number of bioluminescent photons can escape the attenuating environment, and they can be detected using a highly sensitive charge-coupled device (CCD) camera.\(^3\) In this case, the photons propagation in the biological tissue can be well described by steady-state diffusion equation and Robin-type boundary conditions.\(^6,7\) The BLT principles and solution uniqueness for the BLT problem conditions were studied under some practical conditions.\(^3,5,8\) Some numerical algorithms for BLT were already reported, including the finite element based BLT algorithms,\(^10–14\) BLT in combination with PET (OPET),\(^15\) multispectral bioluminescence optical tomography,\(^16,17\) and BLT method based on diffusion theory of half infinite medium.\(^18\) In this letter, for the first time we develop a BLT algorithm using the boundary integral method. This methodology only requires finite element meshing of structural boundaries instead of complex volumetric finite elements previously used for BLT. Hence, the complexity and stability of the new BLT algorithm are improved as compared to that of the finite-element-based BLT algorithms.

The overall support region of a small animal \(\Omega\) can be decomposed into a number of subregions \(\Omega_j\) \((j=1,2,\ldots,\tau)\) with \(\Omega=\sum_{j=1}^{\tau} \Omega_j\); for example, lungs, heart, liver, bone, muscle, and so on. Applying the Gauss theorem, the steady-state diffusion equation can be transformed to a boundary integral equation on each subregion:\(^19\):

\[
\frac{1}{2} \Phi(\mathbf{r}) + \int_{\partial \Omega_j} \left[ \Phi(x) D \frac{\partial G(r,x)}{\partial n} - G(r,x) D \frac{\partial \Phi(x)}{\partial n} \right] dx = \int_{\Omega_j} S(x) dx, \quad \mathbf{r} \in \partial \Omega_j
\]

where \(G(r,x)=\exp(-\mu_{eff}(r-x))/(4\pi D |r-x|)\) is a Green function of the steady-state diffusion equation, \(D\) is the diffusion coefficient given by \(D=1/(3(\mu_a+\mu'_s))\), and \(\mu_{eff}=[3(\mu_a+\mu'_s)]^{1/2}\) where \(\mu_a\) is the absorption coefficient \((\text{mm}^{-1})\) and \(\mu'_s\) is the reduced scattering coefficient \((\text{mm}^{-1})\) for the subregion \(\Omega_j\). Equation (1) is a well-posed second kind integral equation. The boundary surface \(\partial \Omega_j\) can then be split into \(N\) surface elements \(\Gamma^i_j\) \((i=1,2,\ldots,N)\) on which the function \(\Phi(x)\) and \(\partial \Phi(x)/\partial n\) are approximated by use of a set of \(p\) interpolation points and interpolation functions \(\phi_i(x)\) on \(\Gamma^i_j\):

\[
\begin{align*}
\Phi(x) &= \sum_{i=1}^{p} \phi_i(x_i) \phi_i(x), \\
\frac{\partial \Phi(x)}{\partial n} &= \sum_{i=1}^{p} \frac{\partial \phi_i(x)}{\partial n} \phi_i(x)
\end{align*}
\]

The shape of the surface element can be arbitrary selected, and usually made as a quadrilateral or a triangle. The number of points per surface element depends on the accuracy needed in the interpolation procedure. Let \(\Phi_2=\Phi(x_i)\) and \(\phi_i=D \phi_i(x_i)/\partial n\) represent the values of the function \(\Phi(x)\) and \(D \partial \Phi(x)/\partial n\) at an interpolation point \(x_i\), respectively. Since the BLT reconstruction is underdetermined and ill-posed, we incorporate a priori knowledge to define a permissible source region \(\Omega_j^1(\Omega_j^1 \subseteq \Omega_j)\) for a subregion \(\Omega_j\) without loss of generality, where the bioluminescence source may be distributed. The region \(\Omega_j^1\) can be discretized, and the embedded source function \(S(x)\) is approximated on \(\Omega_j^1\) as

\[
S(x) = \sum_{i=1}^{N_i} s_i \psi_i(x),
\]

where \(s_i=S(x_i)\) represent the value of the source function \(S(x)\) at an interpolation point \(x_i\), \(\psi_i(x)\) is the interpolation function, and \(N_i\) is the number of discrete values of the source function.

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Inserting Eqs. (2) and (3) into Eq. (1), we obtain the following matrix equation:

\[
\left( \frac{1}{2} I + M \right) \Phi = H Q + F/S
\]

(4)

where \( \Phi = (\Phi_1, \Phi_2, \ldots, \Phi_M)^T \), \( Q = (q_1, q_2, \ldots, q_M)^T \), \( M \) is the number of nodal points for \( \partial \Omega_j \), \( S = (s_1, s_2, \ldots, s_N)^T \), and \( H_j \) is a strictly diagonally dominant matrix, and allows its inversion. Multiplying Eq. (4) with the inverse \( H_j^{-1} \), we have

\[
H_j^{-1}\left( \frac{1}{2} I + M \right) \Phi = Q + H_j^{-1} F/S.
\]

(5)

Furthermore, in terms of the continuity of \( \Phi(x) \) and \( D \partial \Phi(x) / \partial n \) at the interface and the Robin-type external boundary condition, the above matrix equation for \( j = 1, 2, \ldots, \tau \) can be assembled into

\[
M \Phi = FS,
\]

(6)

where \( S \) denotes the source distribution in the permissible source region, and \( \Phi \) a vector consisting of photon density values at the boundary and interface nodes, which can be divided into \( \Phi^m \) at internal interface nodes and \( \Phi^e \) at exterior boundary nodes. Because \( M \) is still a diagonally dominant matrix and invertible, we have \( B = M^{-1} F \). After removal of those rows of \( B \) that correspond to \( \Phi^m \) we obtain \( B^e \) and a linear relationship between measurable photon density at boundary nodes and the source distribution

\[
\Phi^e = B^e S.
\]

(7)

Generally, the measured data in bioluminescence imaging are corrupted by noise, so it is not practical to solve for \( S \) directly from Eq. (7). Instead, an optimization procedure is employed to find a solution by minimizing the following objective function:

\[
\min_{0 \leq S \leq u} \| \Phi^e - B^e S \|_W + \alpha \eta(S),
\]

(8)

where \( u \) is the upper bound that is chosen to be physically meaningful, \( \eta(S) \) a stabilizing function, \( \alpha \) the regularization parameter, \( W \) the weight matrix, and \( \| V \|_W = V^T W V \).

To demonstrate the feasibility and efficiency of our new algorithm, we carried out an experiment using a heterogeneous mouse chest phantom. The physical phantom of 30-mm height and 30-mm diameter was fabricated. It consisted of four types of high-density polyethylene materials: (8624K16), nylon 6/6 (8538K23), delrin (8579K21), and polypropylene (8658K11) (McMaster-Carr Supply Company, Chicago, Illinois), to represent muscle (M), lungs (L), heart (H), and bone (B) respectively. Based on the diffuse model of photon propagation, the optical parameters of the four materials at wavelength of about 650 nm were independently found from transillumination experimental data, which are listed in Table 1.

Table 1 Optical parameters of the heterogeneous mouse chest phantom.

<table>
<thead>
<tr>
<th>Material</th>
<th>Muscle (M)</th>
<th>Lung (L)</th>
<th>Heart (H)</th>
<th>Bone (B)</th>
</tr>
</thead>
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<tr>
<td>( \mu_c ) [mm(^{-1})]</td>
<td>0.007</td>
<td>0.023</td>
<td>0.011</td>
<td>0.001</td>
</tr>
<tr>
<td>( \mu_s ) [mm(^{-1})]</td>
<td>1.031</td>
<td>2.000</td>
<td>1.096</td>
<td>0.060</td>
</tr>
</tbody>
</table>

Fig. 1 Heterogeneous mouse chest phantom. (a) A geometrical model of the phantom embedded with two sources; (b) a middle cross-section through two embedded hollow cylinders for hosting luminescent sources in one lung.
The reconstruction results may be further improved by decreasing the boundary element size, lowering the measurement noise, and increasing the accuracy of optical parameters. The method only requires the discretization on the object boundary and structural interface, which is much easier than meshing for volumetric finite elements. Hence, the new method can handle a complex geometrical model much efficiently than finite element based BLT algorithm.

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References