BOOK REVIEWS

Ten Lectures on Wavelets

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Science is not immune to the whims of fashion, and one of the current chic topics is wavelets. One author recently claimed that one of the most important mathematical contributions of the second half of the twentieth century is wavelets. As a result, there has been a tremendous outpouring of research papers that purport the use of wavelets. Many researchers see near mystical aspects in wavelets, yet in some cases the term seems only to appear in the title and is vaguely referred to in the introduction. While only time will tell if wavelets live up to their press notices, sophisticated mathematical treatments of wavelets are starting to appear. One of these is *Ten Lectures on Wavelets* by Ingrid Daubechies, a pioneer in this field.

I have read this book from two perspectives, one as a mathematician and the other as a scientist attempting to apply wavelets to my work. The book succeeds more from my perspective as a mathematician than from my perspective as a user. The author acknowledges that this is a mathematical text, and I agree. Clearly, it is important for those expecting to utilize wavelets to understand the mathematical richness that supports them. However, a lecture on implementations and fast wavelet transforms and the like would be useful to those hoping to exploit wavelet potential.

There are several reasons why wavelets are appealing: one is the ability to have good time and frequency localization and another is multiresolution analysis. Multiresolution analysis has shown a close coupling of functional analysis and signal processing. This coupling has been more apparent in the past few years. My engineering colleagues are sounding more like mathematicians as I hear discussions of functional basis functions, convergence properties, and Hilbert spaces. As a mathematician, I find this quite satisfying, and the introduction of wavelets into this community is a welcome event.

There can only be a strengthening for both communities.

Ten Lectures on Wavelets is arranged in ten chapters, one for each "lecture." If anyone is thinking of using this as a basis for a course, don't let the title concern you, plenty of material exists to fill lectures beyond the tenth class. The ten lectures are clearly geared to a very knowledgeable group of listeners. The text would make an excellent choice for a combined graduate-level course in mathematics and electrical engineering.

The material covered is extremely recent; the author references papers published in 1992. There is also an extensive reference list with contributions from a variety of sources. Those interested in pursuing this topic will find this an excellent source of current research topics and applications.

Some minor areas could be improved. One is the discussion of reproducing kernel Hilbert spaces. I had to scan through several texts on functional analysis before I found the definition of these spaces. Moreover, it was not immediately apparent to me why these spaces were of any particular interest. It would be helpful if the definition and some motivation were included in the text. In practice these spaces are precisely the functional domain used by researchers in digital electronic imaging!

I also found some of the figures to be confusing, and for some, I spent a long time trying to determine what the point was for including them. Perhaps a better description or a better drawing would have made that clear. Once I understood the point of the figures, I did find them illuminating.

The book does introduce concepts that are nontraditional, and then uses them only for a short period. I found myself spending a fair amount of time trying to understand the value of notions such as the Zak-transform. I had not previously encountered the Zak-transform and went on a small detour trying to understand and appreciate the utility and point of this device. Unfortunately, the reward for the effort was not commensurate with the effort. While a significant theorem was proven by using the Zak-transform, it might have been advantageous to rely on more traditional tools, though not as elegant for the task.

The author is quite adept at showing the utility of wavelets for a number of applications such as image processing and data compression. For several pages the author describes QMF and subband coding in the most lucid manner I have ever seen. More

applications like this would be a great asset. Because one can consider wavelets as a means of generalizing Fourier transforms and improving windowed Fourier transforms, the utility of wavelets to imaging is obvious. In fact, much of the motivation behind wavelets is from electronic imaging.

While the material on frames and convergence properties in the early chapters is quite important, the material of practical interest does not appear until the fifth chapter, which is the beginning of the treatment on multiresolution analysis. Many of the concepts in this chapter have appeared in various forms over the years, e.g., quadrature mirror filters and subband coding. Building an elegant mathematical theory to support these approaches is extremely valuable. Moreover, a method to construct wavelets for specific applications is exceedingly helpful. The question then becomes one of finding the proper mother wavelet or best-suited scaling function for a given application. The mathematics developed to proceed from there are solid and extremely elegant. If for no other reason, this book should be read for this chapter alone.

A good part of the second half of the book is concerned with compactly supported wavelets. The ability to determine compactly supported wavelets is indeed important. One of the most famous examples of this is the Haar transform, which is largely the foundation of using wavelets for data compression. However, selecting compactly supported wavelets must be done with care to maintain good frequency localization. Having a theory to help in designing wavelets that have both properties is extremely valuable.

I heartily recommend this book to anyone who seriously wants to know the state of the art in wavelets. The modest cost of thirty dollars is a real bargain for the wealth of information it contains. I also recommend to those without a firm foundation in functional analysis to find someone who has the knowledge to help in reading and understanding this work. This book is a pleasure to read, although, trying to read it without the necessary mathematical knowledge could be a frustrating experience. I have changed from one who is skeptical because of wavelets being fashionable to one who has the opinion that wavelets are here to stay and will become a standard tool in many applications. This book is likely to become a classic text in mathematics and a reference to those seriously using wavelets. It is exciting to read a soon-to-be classic.