Importance of boundary reflections in the theory of diffusive light scattering

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In a recent paper, McMurry, Weaire, Lunney, and Hutzler\(^1\) claimed that the angular dependence of exiting photons diffusively transmitted through a disordered multiple-light-scattering material is controlled by scattering anisotropy in terms of the ratio \(\ell_*/\ell_s\) of the transport to scattering mean free paths. Motivated by discussion with Weaire, I considered the same problem but reached a different conclusion.\(^2\) Namely, the angular dependence is approximately independent of scattering anisotropy and depends strongly on the reflectivity of the sample boundary. Within the confines of a diffusion approximation, transport is best described by the photon concentration field \(U(r)\) satisfying a diffusion equation with \(D=(1/3)\ell_*/\ell_s\) and boundary conditions such that \(U(r)\) extrapolates to zero at distance \(z_e\times\ell_*/\ell_s\) outside the sample. The value of the extrapolation length ratio \(z_e\) is chosen so that the fictitious flux of photons entering the sample equals the boundary reflectivity times the flux leaving. This gives \(z_e=(2/3)(1+R_2)/(1-R_1)\) where \(R_n=(n+1)\int_0^{\mu_s} R(\mu)\,d\mu\) and \(R(\mu)\) is the total reflection probability for a photon striking the sample boundary at angle \(\cos^{-1}\mu\) with respect to the normal. Given such a concentration field, the angular dependence of the exiting photons can be found by straightforward kinetics. Ignoring refraction, I calculated that the probability \(P(\mu)\,d\mu\) for a transmitted photon to exit between \(\cos^{-1}\mu\) and \(\cos^{-1}(\mu+d\mu)\) from the normal is given by

\[
P(\mu)/\mu = \frac{z_e + \mu}{z_e/2 + 1/3}.
\]

It thus contains a mixture of cosine and cosine-squared dependence that depends on the boundary reflectivity through the value of \(z_e\).

In Ref. 2 I tested Eq. (1) by comparison with random walk computer simulations incorporating both scattering anisotropy and boundary reflectivity. Walkers were launched from one edge of a slab with thickness \(L=15\ell_*/\ell_s\) and allowed to wander according to the values of \(\ell_*/\ell_s\) and \(R(\mu)\) until they exited at either edge. Figure 1 shows new simulation data for isotropic scattering and several constant boundary reflectivities \(R(\mu)=R\). Evidently, \(P(\mu)/\mu\) varies dramatically with \(R\) but is nearly linear in \(\mu\) and compares quite well with the prediction of Eq. (1).

When anisotropy is included, the angular dependence is unaffected except for a subtle dip at glancing angles (see Fig. 5 of Ref. 2). As demonstrated in Fig. 2, the mixture of cosine and cosine-squared dependence near the forward direction is still controlled by boundary reflectivity. Results for \(z_e\) based on Eq. (1), multiplied by a new normalization factor, fit to simulation data for walkers exiting within 45 deg of the normal are shown by solid symbols versus \(\ell_*/\ell_s\). To within statistical uncertainty, these values of \(z_e\) are constant and equal to the prediction \((2/3)(1+R)/(1-R)\). If the data are instead fit to Eq. (1) over the entire range of \(\mu\), including the
dip near glancing angles where \( P(\mu)/\mu \) deviates from linearity, then the resulting values of \( z_* \) decrease systematically with \( \ell^*/\ell_{s} \), as shown by the open symbols, but have little significance since the functional form is incorrect. This could explain the increase in cosine-squared dependence with increasing \( \ell^*/\ell_{s} \) obtained by McMurry et al. from fits to their own simulation data. Perhaps the theoretical ideas they advance could be used to quantitatively explain the slight discrepancy at small \( \mu \) between simulation results and Eq. (1) in terms of the scattering anisotropy. However, it should be cautioned that such deviations can be small compared to refraction effects and may depend on more details of the scattering form factor than just the value of \( \ell^*/\ell_{s} \).

In conclusion, the angular dependence of diffusely transmitted light is set primarily by boundary effects independent of scattering anisotropy. This is also born out by experimental work on suspensions of polystyrene spheres of variable size, where refraction and polarization are also important. Scattering anisotropy has at most a subtle influence on behavior at glancing angles; there, detected photons originate in a region very close to the boundary where diffusion approximations are least accurate. Contrary to the conclusion of Ref. 1, the functional form of \( P(\mu) \) thus tells little about the structure of the scattering material itself. Rather, its importance is in revealing the nature of the sample boundary and in showing what value of \( z_* \) should be used in diffusion theory predictions for the transmission probability and the diffusing-wave spectroscopy autocorrelation function. With proper choice of \( z_* \), accuracies on the order of 1% can be obtained without recourse to numerical solution of the exact transport equations.

References


Response to “Importance of boundary reflections in the theory of diffusive light scattering”

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In a recent paper we addressed the problem of the angular dependence of light transmitted through a foam, which is predicted by the diffusion model in the case of slab geometry to be of the form

\[
T(\theta) = a \cos \theta + b \cos^2 \theta ,
\]

where \( \theta \) is the angle the transmitted light makes with the normal to the exit face, and \( b/a = 3/2 \) if the usual boundary condition, that there is no net inward flux of diffuse light at the exit face, is applied. Measurements on real foam samples indicated higher values of \( b/a \), and random walk simulations showed that this ratio increases with the degree of anisotropy of the local scattering (measured by the ratio \( \ell/\ell^* \) of the mean free path to the transport mean free path).

Durian has called these results into question on the following grounds:

1. The angular distribution of the transmitted light is strongly affected by reflection at the boundary, when a glass container is used.
2. His random walk simulations showed no dependence on anisotropy.

We fully agree with Durian on point 1. However, this problem did not arise in our measurements on solid foam, which needs no container. Our random walk program, with no reflection at the boundary, is therefore an appropriate simulation for these measurements. These simulations showed a variation of \( b/a \) that was produced only by the anisotropy of the local scattering.

Our experimental results for liquid foam, in a glass container, indicated that \( b/a \) was larger than 3/2. Our simulations showed such an increase in the case of predominantly forward local scattering. However, incorporating the effects of reflection through introducing an average reflectance \( R \) in the diffusion model decreases, rather than increases, the ratio \( b/a \), giving \((3(1 - R))/(2(1 + R))\) instead of 3/2.

With regard to point 2 we have two comments to make in reply. First, Durian uses an exponential step-length distribution in his simulations, in contrast to the fixed step-length used in Ref. 1. Our simulations using the exponential step-length distribution show a reduction in the range over which \( b/a \) varies compared to the case of fixed step length. This is shown in Fig. 1. However, an increase in \( b/a \) as \( \ell/\ell^* \) decreases is still apparent. (The scattering function used in Ref. 3, and in the results displayed in Fig. 1, is equally anisotropic for all values of \( \ell/\ell^* \), since it is essentially a delta function, selecting a particular value of the scattering angle. In particular, it does not approach isotropic scattering as \( \ell/\ell^* \to 1 \). However, the data we obtained with it show similar trends to that which we obtained using the scattering function of Ref. 1 or the Heneyy-Greenstein function.)

Second, the variation of \( b/a \) with \( \ell/\ell^* \) is somewhat obscured if \( [T(\theta)]/\cos \theta \) is plotted as a function of \( \cos \theta \) as in Ref. 3. If the simulation data used to produce Fig. 1 are replotted in this way the result is similar to the case \( R = 0 \) in Fig. 5 of Ref. 3. We have used these simulation data to calculate \( b/a \) in two different ways for each of five different values of \( \ell/\ell^* \):

OPTICAL ENGINEERING / November 1995 / Vol. 34 No. 11 / 3345
Fig. 1 The variation of $b/a$ with $\ell/\ell^*$ from simulations using the scattering function of Ref. 3, with a fixed step-length (●) and the step-length distribution of Ref. 3 (○). The points (○) and (×) refer to isotropic scattering in the case of a fixed and distributed step length, respectively.

Table 1 Simulation data.

<table>
<thead>
<tr>
<th>$\ell/\ell^*$</th>
<th>$b/a$, linear fit</th>
<th>$b/a$, quadratic fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>1.42 ± 0.04</td>
<td>1.48 ± 0.07</td>
</tr>
<tr>
<td>0.3</td>
<td>1.96 ± 0.06</td>
<td>1.93 ± 0.08</td>
</tr>
<tr>
<td>0.1</td>
<td>2.94 ± 0.15</td>
<td>2.15 ± 0.10</td>
</tr>
<tr>
<td>0.03</td>
<td>3.02 ± 0.15</td>
<td>1.95 ± 0.08</td>
</tr>
<tr>
<td>0.01</td>
<td>3.37 ± 0.18</td>
<td>2.18 ± 0.10</td>
</tr>
</tbody>
</table>

1. by finding the best linear fit to $[T(\theta)]/\cos \theta$ as a function of $\cos \theta$
2. by finding the best quadratic fit to $T(\theta)$ as a function of $\cos \theta$.

The results are given in Table 1, and an example of the two fits, for $\ell/\ell^* = 0.1$, is shown in Fig. 2. It is clear that method 1 gives a poorer fit to the data, probably due to magnification of small errors introduced by dividing the intensity $T(\theta)$ by $\cos \theta$ for $\cos \theta$ close to zero. Even the linear fit, however, shows a definite increase of $b/a$ as $\ell/\ell^*$ decreases.

In conclusion, we believe that $b/a$ does depend on the degree of anisotropy of the local scattering, although the variation is not large, particularly in the case of an exponentially distributed step-length. Clearly, reflection at the container walls affects experimental measurements of the angular distribution of the transmitted light, but our investigations concentrated on first establishing the effects associated with the intrinsic structure of the foam, since it is important to be able to distinguish between these and effects due to reflection.

**References**
Comment on the paper
“Elimination of systematic error in subpixel accuracy centroid estimation”

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1 Introduction

In a paper published in 1991, Alexander and Ng\(^1\) introduced an excellent analysis of the systematic error in centroid estimation. They used Fourier analysis to develop a closed-form equation for estimating the centroid of a shifted symmetric function. However, as we will show here, the tools they developed can be extended to address the general case of any real signal.

2 Centroid of Continuous Real Signal

Consider a continuous real signal \(f(s)\). Its Fourier transform \(F(s)\) is given by

\[
F(s) = F_e(s) \exp\{j \phi_e(s)\}.
\]

(1)

The magnitude of \(F(s)\), namely \(F_e(s)\), is an even function of the frequency \(s\), and its phase \(\phi_e(s)\) is an odd function. If \(f(s)\) is shifted by an amount \(d\), the Fourier transform is given by

\[
F(s) = F_e(s) \exp\{j [\phi_e(s) - 2\pi ds]\}.
\]

(2)

Its derivative \(F'(s)\) is given by

\[
F'(s) = F_e'(s) \exp\{j [\phi_e(s) - 2\pi ds]\}
+ F_e(s) j [\phi_e'(s) - 2\pi d]
\times \exp\{j [\phi_e(s) - 2\pi ds]\}.
\]

(3)

Thus, we have

\[
F(0) = F_e(0) \exp\{j [\phi_e(0) - 2\pi d(0)]\}
= F_e(0).
\]

(4)

and similarly

\[
F'(0) = F_e(0) j [\phi_e'(0) - 2\pi d].
\]

(5)

Therefore, the Fourier transform representation of the centroid is given by\(^2\)

\[
c = \frac{F'(0)}{-2\pi j F(0)}
= d - \frac{\phi_e'(0)}{2\pi}.
\]

(6)

The first term in Eq. (6) is the amount of shift applied to the signal. The second term is a phase shift that accounts for the centroid location of the unshifted real signal.

3 Centroid of Sampled Real Signal

When the signal \(f(s)\) is sampled at a sampling rate \(T\), the Fourier transform of the discrete signal is given by

\[
G(s) = \sum_{-\infty}^{+\infty} F\left(s - \frac{n}{T}\right)
= \sum_{-\infty}^{+\infty} F_e\left(s - \frac{n}{T}\right)
\times \exp\left\{j \left[\phi_e\left(s - \frac{n}{T}\right) - 2\pi \left(s - \frac{n}{T}\right)d\right]\right\}
\]

(7)

and thus

\[
G'(s) = \sum_{-\infty}^{+\infty} F_e'\left(s - \frac{n}{T}\right)
= \sum_{-\infty}^{+\infty} F_e'\left(s - \frac{n}{T}\right)
\times \exp\left\{j \left[\phi_e\left(s - \frac{n}{T}\right) - 2\pi \left(s - \frac{n}{T}\right)d\right]\right\}
\]

(8)

Evaluating Eqs. (7) and (8) for \(s = 0\), and rearranging terms, we have

\[
G(0) = F_e(0) + 2 \sum_{1}^{\infty} F_e\left(\frac{n}{T}\right)
\times \cos \left[\phi_e\left(\frac{n}{T}\right) - 2\pi \left(\frac{n}{T}\right)d\right]
\]

(9)
and

\[ G'(0) = F_e(0) j[\phi'_e(0) - 2\pi d] \]

\[ + 2j \sum_{n=1}^{\infty} F_e' \left( \frac{n}{T} \right) \sin \left[ \phi_e \left( \frac{n}{T} \right) - 2\pi \frac{n}{T} d \right] \]

\[ + 2j \sum_{n=1}^{\infty} F_e' \left( \frac{n}{T} \right) \left[ \phi'_e \left( \frac{n}{T} \right) - 2\pi d \right] \times \cos \left[ \phi_e \left( \frac{n}{T} \right) - 2\pi \frac{n}{T} d \right] \]  

(10)

Finally, the centroid of the sampled signal is given by

\[ \bar{c} = \frac{G'(0)}{-2\pi j G(0)} \]

\[ = d - \frac{\phi'_e(0)}{2\pi} \]

\[ - \frac{\sum_{n=1}^{\infty} F_e' \left( \frac{n}{T} \right) \sin \left[ \phi_e \left( \frac{n}{T} \right) - 2\pi \frac{n}{T} d \right]}{\pi \left( F_e(0) + \sum_{n=1}^{\infty} F_e' \left( \frac{n}{T} \right) \cos \left[ \phi_e \left( \frac{n}{T} \right) - 2\pi \frac{n}{T} d \right] \right)} \]  

(11)

A first-order approximation of the centroid systematic error is obtained by using the first term in the numerator and neglecting the sum in the denominator relative to \( F_e(0) \), which results in

\[ \bar{c} = c_{ms} = \frac{F_e' \left( \frac{1}{T} \right) \sin \left[ 2\pi \frac{d}{T} - \phi_e \left( \frac{1}{T} \right) \right]}{\pi F_e(0)} \]  

(12)

Thus, as expected, the general expressions for the centroid locations, and centroid estimation error for a real signal, are similar to the equations derived for the special case of shifted symmetric signal, with additional terms accounting for the nonzero phase.

References

Acousto-Optic Signal Processing: Fundamentals and Applications

Reviewed by Mark D. McNeill, Bucknell University, Department of Electrical Engineering, Lewisburg, PA 17837.

In the preface of this text, the authors state the following: “We hope that this book will introduce both the graduate student and the practicing engineer to the usefulness of acousto-optic techniques, and re-introduce many approaches for the reference of the practicing engineer.” To this end, this volume provides a survey of the more fundamental aspects of acousto-optics while also including recent applications to modern problems. Except for the first short historical review chapter, each chapter has worked examples and end-of-chapter problems. Also, the references given at the end of each chapter provide a thorough list of original works for the interested reader. However, some of these lists are simply overwhelming. For example, Chap. 5 provides over 150 references.

The treatment of the subject generally follows a logical path. Chapters 1, 2, and 3 provide the necessary background for the reader new to this field. The introductory chapter provides a historical review of acousto-optics and cites many important contributors to this field. It brings the reader quickly into the modern-day applications that link this field to current and future technologies.

The second chapter uses a standard phase-grating approximation theory to qualitatively explain acousto-optic interaction in isotropic media. Different acousto-optic figures of merit are derived here, as are the Raman-Nath parameter, the Klein-Cook parameter, the Raman-Nath regime, and the Bragg regime.

The third chapter begins with a short overview of crystals and piezoelectric material and gives examples of diffraction in isotropic media. It then presents a coupled-mode theory, polarization effects, anisotropic Bragg interaction, optical activity, and photorefractive effects. Imbedded in this chapter is a rigorous diffraction theory that begins with Maxwell’s equations and rederives more or less the coupled-mode equations in an earlier section. Often I felt that there was not enough information about the proposed theory, and the jumps to final solutions were too abrupt. Although there is great effort to present this in a self-contained format, constant referrals to the references make the chapters difficult to follow at times. Chapter 4 then presents a detailed review of surface acoustic wave (SAW) devices. This chapter begins with a basic SAW device and develops the accompanying interaction theory. There is much information packed into this chapter, thus making it one of the most interesting in the text.

Chapters 5 and 6 follow the format presented in the latter part of Chap. 4. The main purpose of Chap. 5 is to introduce the reader to a variety of designs proposed in literature elsewhere. The sections in the chapter are concise, however, each device is accompanied by an illustration and a mathematical theory of operation. Some topics include mode locking and Q-switching, tunable filters, spectrum analyzers, acousto-optic modulators, deflectors, convolvers and correlators, and heterodyne processing. Chapter 6 uses the building blocks of Chaps. 2 through 5 to build more complex systems with applications to optical pattern recognition, spread spectrum communications, and adaptive signal processing. It follows the style of Chap. 5 except that more explanatory text is provided to help the reader. Many interesting applications of acousto-optic signal processing are included in this section.

Lastly, Chap. 7 investigates possible areas of continuing research such as bistable devices, acousto-electro-optic effects, and optical neural networks.

What I liked most about this text is the very large number of applications presented by the authors. There are not enough pages to cover these areas in great depth, however, sufficient text, figures, and references clearly help to engage the reader. The authors also seem to always present an application with an accompanying acousto-optic theory. This allows the reader to acquire a fundamental understanding of acousto-optics and a general appreciation for the many systems that can benefit from acousto-optic techniques.

What I liked least about the book is the theoretical review sections. While I did not attempt to verify every equation in the text, I found enough errors in Chap. 2 to make me suspicious about the rest of the text. At best these errors unnecessarily confuse the reader new to this subject matter. Also, throughout these sections the authors constantly switch the relative directions of light and sound, which cause some figures not to match the text that explains them.

I would recommend this book to the serious student who wants to know more about acousto-optic signal processing applications. It may also serve as a text for an introductory graduate course about acousto-optics, although the instructor should be aware of the errors mentioned above. The practicing engineer or researcher may also find the book useful because of its many applications and exhaustive reference lists.
High-Power Optically Activated Solid State Switches


Reviewed by Roger A. Dougal, Department of Electrical and Computer Engineering, University of South Carolina, Columbia, SC 29208.

What is a high-power system, and what is the role of a switch in such a system? That is the question first answered in this well-written, multiauthored book, as Buttram conveys the reader into the pulsed-power community to better understand the need for and benefits of optically activated switches. Brief descriptions are provided for electrical impulse generators designed as electron accelerators, microwave generators, and nuclear effects simulators. Examples such as the superpower Saturn machine, which generates 40-ns impulses of 2.5 TW peak power, illustrate the magnitude of the challenge. The fundamental principle of operation of these machines, accumulating energy in a storage device then releasing it quickly by toggling a switch, is explained in such a way that a diverse audience can appreciate the nature of the inherent problems.

Born with the advent of radar, the pulsed-power community is still vital some 60 years later, but continues to seek a better electrical switch, one that offers the ability to accurately control the delivery of very high power impulses. The most common switches in pulsed-power systems (spark gaps) are based on a very simple technology — forming an arc between two metal electrodes. Such fundamentally simple switches are the stock devices in high-power systems, yet they are not adequately controllable for many applications. Ionization times are too long, recovery rates are too slow, and the ability to turn off a flowing current is almost nonexistent. Optically activated semiconductor switches promise to overcome these obstacles and to deliver all of the desirable characteristics that ordinary switches do not deliver.

In Chap. 2, Nunnally describes the operation of photovoltaic power switches that operate in the linear mode — each electron-hole pair being created by one photon. Such switches allow precise control of high-power impulses, with the closure time being limited only by the intensity of the light source (generally a laser). In principle, such switches can also open, either by recombination of the charge carriers or by sweeping of the carriers out of the high field region. Optical activation always requires timing of closure so that multiple switches can be synchronized to almost arbitrary precision.

Pocha and Hofer next describe high-speed switching in photoconductive switches and introduce the reader to the concept of the high-gain switch, in which the number of activated charge carriers exceeds the number of photons introduced into the switch. Switching properties of materials such as GaAs and diamond are described in this context. Applications of photoconductive switching in microwave generation are described.

Several authors of Chap. 4 explain the operation of inductive energy storage systems and the need for opening switches in such systems. The photoconductive switch sometimes can meet that need, but often fails to open when commanded to do so. This “lock-on” feature of the optical switch is explored in some detail.

Persistent conduction is used in another way in the family of copper-doped GaAs switches described by Schoebach. Here, conduction is initiated by illuminating the switch with a particular wavelength of light, then terminated by illuminating with another color of light. This makes a fully controllable switch — one that can be turned on and off — and one that potentially has many applications. This switch still suffers from performance limits that prevent it from being used at even moderate powers, but the possibility for very precise control of electrical power makes it an attractive candidate for additional research.

Optical activation of high-power switches requires high-power light sources; almost universally these are lasers. In some cases, especially in the high-gain mode of operation, the laser that activates conduction can be quite small. Even semiconductor diode lasers can fill the bill, allowing for very compact source-switch configurations. Other switches require higher powers from the laser than diode lasers can currently deliver, but research is underway to increase the power and energy available from semiconductor lasers so that optically activated switches can be as compact and reliable as possible. The current state of the art in high-power diode laser source development is described in Chap. 7, including edge-emitting and surface-emitting lasers and laser arrays. Optical engineers will particularly appreciate this chapter, as it illustrates the aspects of the source problem that are important to pulsed-power engineers.

The remaining chapters of the book deal with current filamentation in high-gain switches, general research issues in photoconductive switching, and special geometries of optically activated switches, including MEFET and thyristor structures. At this point, the reader begins to appreciate that substantial additional effort must be...