

Partial Polarization: A comprehensive student exercise

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Abstract

We present a comprehensive student exercise in partial polarization. Students are first introduced to the concept of partial polarization using Fresnel Equations. Next, MATHCAD is used to compute and graph the reflectance for dielectrics materials. The students then design and construct a simple, easy to use collimated light source for their experiment, which is performed on an optical breadboard using optical components typically found in an optics lab above the introductory level. The students obtain reflection data that is compared with their model by a nonlinear least square fit using EXCEL. Sources of error and uncertainty are discussed and students present a final written report. In this one exercise students learn how an experiment is constructed “from the ground up”. They gain practical experience on data modeling and analysis, working with optical equipment, machining and construction, and preparing a final presentation.

Keywords: Fresnel equations, data modeling, data analysis, polarization

1. Introduction

Students are typically introduced to the Fresnel equations in commonly used textbooks¹⁻⁴ and in their Electromagnetism and Optics courses. The derivations of these equations from Maxwell’s equations and boundary conditions typically take the form shown in Figure 1, where the electric field is consider either parallel (*p*-polarization) or perpendicular (*s*-polarization) to the plane-of-incidence for dielectric materials. The discussions that usually follow focus on the applications of the Fresnel equations to include the amplitude reflection and transmission coefficients and the reflectance and transmittance with the reflectance as a function of the angle usually presented. However, partial polarization (or the degree of polarization) is often presented in terms of the ratio of the constituent fluxes that are polarized and unpolarized and most textbooks focus on the Brewster angle - its meaning and determination - when further discussing reflectance.

Since partial polarization plays an important role in almost all aspects of reflection we feel that it should be emphasized more and we have created the following laboratory exercise. While there are available introductory laboratory exercises that let students explore polarization and determine the Brewster angle, the emphasis on measuring polarization in terms of partial polarization seems to be lacking. Additionally, these exercises are usually ‘turn-key’ setups where the students merely operate equipment that has already been set up by a laboratory instructor or technician, and all that remains for the student to do is follow the procedure and then record and analyze the data. In the exercise that we present, the student takes a more active role in the experiment. He or she will have to start almost from the “ground up” and perform many tasks that the laboratory instructor or technician would do. We feel this is a better pedagogical approach since it is more in line with what students actually do in an advance laboratory or job setting.

2. Theory

In this section we will not give a detailed derivation of the Fresnel equations but rather a review that is presented in many textbooks. The derivation of the Fresnel equations for the two cases of the electric field parallel and perpendicular to the plane-of-incidence are aided by the diagrams in Figure 1.

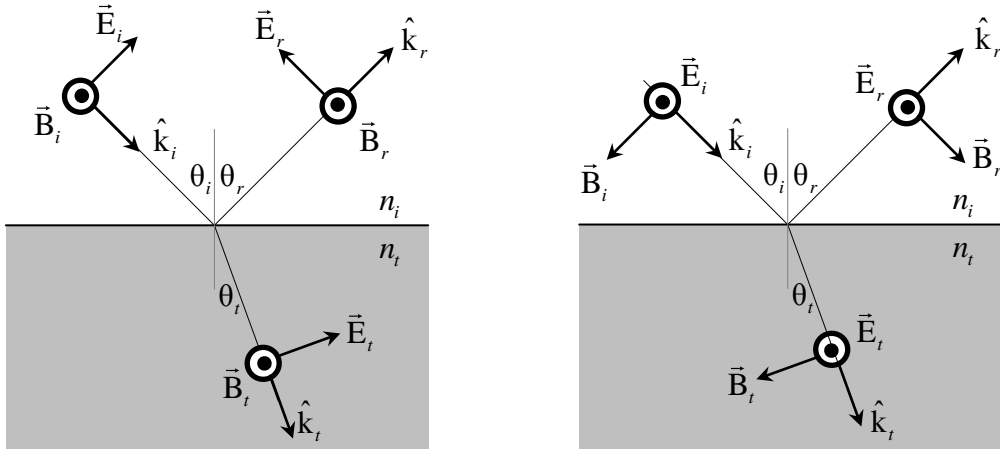


Figure 1. Orientation of the electric and magnetic fields for reflection and transmission of an electromagnetic wave incident on a dielectric medium. The left drawing shows the p - polarization (electric field parallel to the plane of incidence) and the right drawing shows the s - polarization (electric field perpendicular to the plane of incidence).

In the derivation boundary conditions are invoked for non-magnetic dielectric interfaces to arrive at two simultaneous equations that are algebraically manipulated to obtain

$$r_{\perp} \equiv \left(\frac{E_{0r}}{E_{0i}} \right)_{\perp} = \frac{n_i \cos \theta_i - n_t \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} = \frac{\sin(\theta_t - \theta_i)}{\sin(\theta_i + \theta_t)}$$

$$t_{\perp} \equiv \left(\frac{E_{0t}}{E_{0i}} \right)_{\perp} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t} = \frac{2 \sin \theta_t \cos \theta_i}{\sin(\theta_i + \theta_t)}$$

for the case when the electric field is perpendicular to the plane-of-incidence and

$$r_{\parallel} \equiv \left(\frac{E_{0r}}{E_{0i}} \right)_{\parallel} = \frac{n_t \cos \theta_i - n_i \cos \theta_t}{n_i \cos \theta_i + n_t \cos \theta_t} = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}$$

$$t_{\parallel} \equiv \left(\frac{E_{0t}}{E_{0i}} \right)_{\parallel} = \frac{2n_i \cos \theta_i}{n_i \cos \theta_i + n_t \cos \theta_t} = \frac{2 \sin \theta_t \cos \theta_i}{\sin(\theta_i + \theta_t) \cos(\theta_i - \theta_t)}$$

for the case when the electric field is parallel to the plane-of-incidence. When the incident and reflected powers are the same, the reflectance (R) can be defined in terms of the ratio of the incident and reflected flux densities as $R \equiv I_r / I_i$. Since the incident and reflected beams are in the same medium the result can be easily written as

$$R = \left(\frac{E_{0r}}{E_{0i}} \right)^2 = r^2.$$

In similar fashion, when the incident and reflected medium are the same, the transmittance is easily written as

$$T = \frac{n_t \cos \theta_t}{n_i \cos \theta_i} \left(\frac{E_{0t}}{E_{0i}} \right)^2 = \frac{n_t \cos \theta_t}{n_i \cos \theta_i} t^2$$

where the term in front of t^2 reflects the different speeds of the two waves in the two different media and the fact that the cross-sectional area of the incident and transmitted beams are different. It is convention to express the reflectance and transmittance in their component forms $R_{\perp} = r_{\perp}^2$ ($R_{\parallel} = r_{\parallel}^2$)

$$T_{\perp} = \left(\frac{n_t \cos \theta_t}{n_i \cos \theta_i} \right) t_{\perp}^2$$

and

$$T_{\parallel} = \left(\frac{n_t \cos \theta_t}{n_i \cos \theta_i} \right) t_{\parallel}^2$$

where it is readily shown that $R_{\parallel} + T_{\parallel} = 1$ ($R_{\perp} + T_{\perp} = 1$). Since it is the reflectance that is easily measured, and what this laboratory exercise focuses on, we consider only the reflectance equations, which can be written as

$$R_{\parallel} = r_{\parallel}^2 = \frac{\tan^2(\theta_i - \theta_t)}{\tan^2(\theta_i + \theta_t)} \quad (1)$$

$$R_{\perp} = r_{\perp}^2 = \frac{\sin^2(\theta_i - \theta_t)}{\sin^2(\theta_i + \theta_t)}. \quad (2)$$

Figure 2 (left) shows these equations plotted as a function of the incident angle, along with $R = (R_{\perp} + R_{\parallel}) / 2$ when the light is totally unpolarized and the polarization states have the same energy. Likewise, the partial polarizations can be written as

$$\frac{R_{\perp}}{R_{\perp} + R_{\parallel}} \quad (3)$$

$$\frac{R_{\parallel}}{R_{\perp} + R_{\parallel}} \quad (4)$$

and are shown in Figure 2 on the right. It is equations (1) - (4) that students focus on in this laboratory exercise.

3. Experiment

Since the Fresnel equations for partial polarization are derived for unpolarized light incident on a dielectric interface, the first part of the laboratory exercise has the students design and construct a collimated unpolarized light source using the available equipment found in a modern optics laboratory. Students need to first research and learn how a collimated beam is produced and then build one similar to that shown in Figure 3.

Here the light source is a small 2-Volt bulb that has two wires soldered to it, is wrapped in soft foam, and is place inside a metallic cylinder. An aperture, cut from card stock with a small hole, is placed in front of the bulb to reduce its size - making it closer to a point source - and a converging lens is used to collimate the beam. Once students have constructed and adjusted the collimation of the beam, they are ready to design the optical set up which is shown in Figure 4.

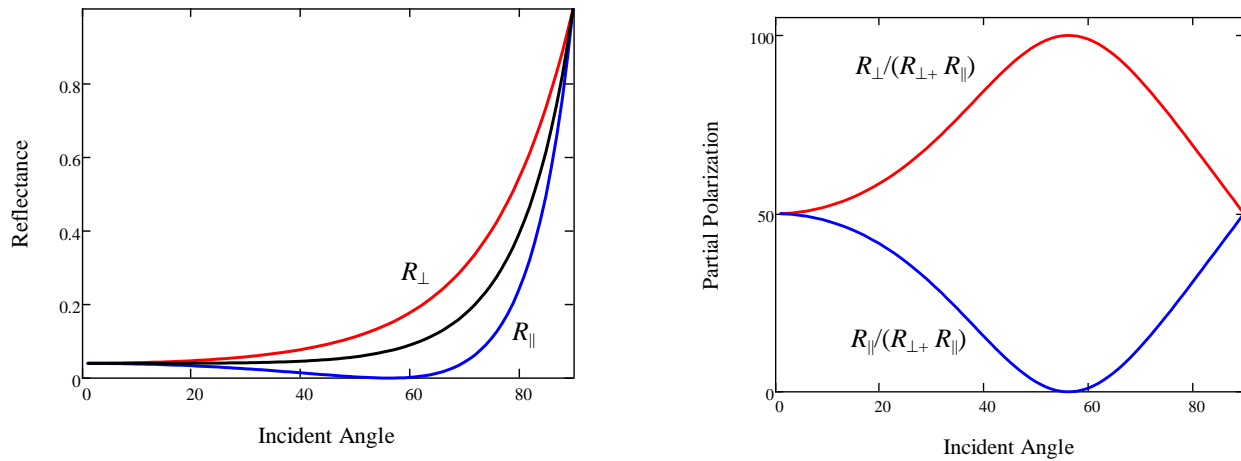


Figure 2. The left figure shows the reflectance R_{\perp} (red), R_{\parallel} (blue), and $R = (R_{\perp} + R_{\parallel}) / 2$ (black) as a function of the incident angle. The right figure is the partial polarization $R_{\perp} / (R_{\perp} + R_{\parallel})$ (red) and $R_{\parallel} / (R_{\perp} + R_{\parallel})$ (blue).

As shown in Figure 4, a glass slide is used as the reflecting surface, but other dielectric materials could be used as well. The slide is mounted on a rotation stage while the polarizer and power meter detector head are mounted on a separate stage. Here the students must figure out the design that allows them to independently rotate the reflecting sample and the detector assembly. This is easily accomplished by using two stacked rotation stages as shown in Figure 5 where the spacing between the two stages is maintained by two $\frac{1}{4}$ - 20 bolts with securing nuts. This configuration is fairly stable, allows the two stages to rotate independently, and - since rotating the polarizer by hand can cause a small movement in the position - the lower stage can be used to accurately position the detector assembly should it be accidentally moved.

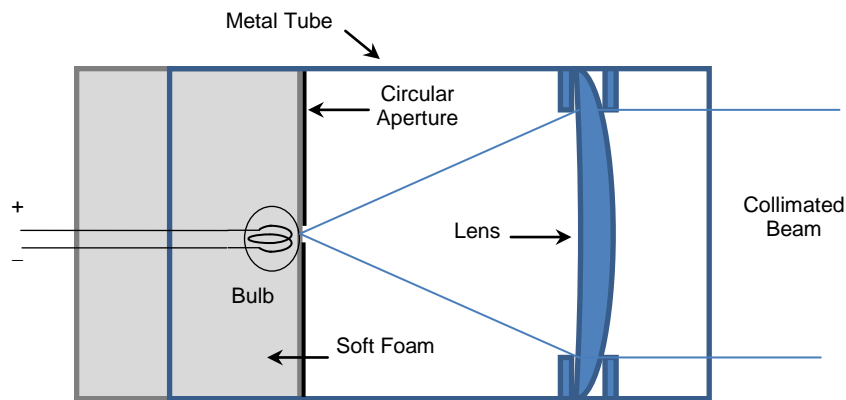


Figure 3. Diagram of a light source used in this experiment.

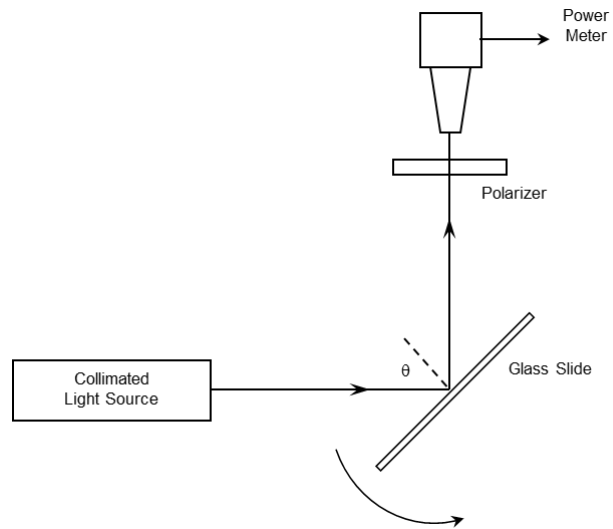


Figure 4. Diagram of the optical components used in the experiment.

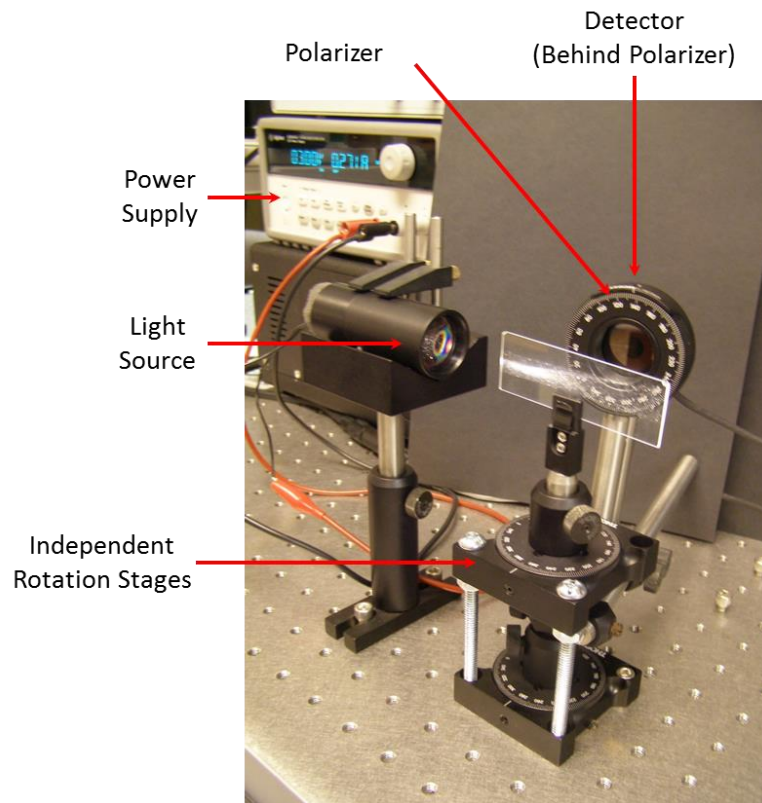


Figure 5. Image of the final experimental assembly with the pertinent parts labeled.

Once the setup is complete and aligned (as well as possible given the collimated light source) students begin taking data. The reflecting stage can be rotated 360° while the detector stage is easily rotated from ~ 20° to < 90° without interference. The polarizer's fiducial setting is vertical, making it perpendicular to the plane-of-incidence. To take data the reflection stage is set at a particular angle and the polarizer is rotated so that measurements are recorded at 0°, 45°, 90°, and 135°. These measurements not only will give the students R_{\parallel} and R_{\perp} but combinations will yield the Stokes parameters³ [$I = I(0^\circ) + I(90^\circ)$, $Q = I(0^\circ) - I(90^\circ)$, $U = I(45^\circ) - I(135^\circ)$] that will allow them to determine the degree of polarization and polarization position angle ζ , given by the equation $\tan(2\zeta) = U / Q$. Repeated measurements at other angles will allow the student to determine the reflectance as well as the partial polarization as a function of incident angle as shown in Figure 2. A typical data set is shown in Figure 6 with the graphs for R_{\perp} , R_{\parallel} and $R = (R_{\perp} + R_{\parallel}) / 2$ shown in Figure 7 (left) and graphs for the partial polarizations in Figure 7 (right).

Polarizer Angle	Experimental												180° w/o pol.
$\theta_i =$	20	30	40	50	52	54	56	58	60	70	75	75	w/o glass
0	192	225	291	399	434	463	521	554	642	1027	1255	1255	3.913
45	152	157	175	212	238	244	274	295	349	649	913	913	3.914
90	136	104	62	17.4	11.4	7.5	5.6	6.8	11.8	179	418	418	3.932
135	176	174	181	198	217	222	258	266	315	536	791	791	3.960
q =	0.171	0.368	0.649	0.916	0.949	0.968	0.979	0.976	0.964	0.703	0.500	0.500	-0.002
u =	-0.073	-0.051	-0.017	0.034	0.046	0.047	0.030	0.052	0.051	0.095	0.072	0.072	-0.006
p =	18.575	37.135	64.894	91.706	94.993	96.927	97.919	97.712	96.526	70.959	50.540	50.540	0.638
$R_{\perp}/(R_{\parallel} + R_{\perp}) =$	58.54	68.39	82.44	95.82	97.44	98.41	98.94	98.79	98.20	85.16	75.01	75.01	
$R_{\parallel}/(R_{\parallel} + R_{\perp}) =$	41.46	31.61	17.56	4.18	2.56	1.59	1.06	1.21	1.80	14.84	24.99	24.99	
$R_{\perp}(\%) - R_{\parallel}(\%) =$	17.07	36.78	64.87	91.64	94.88	96.81	97.87	97.57	96.39	70.32	50.03	50.03	
$R = (R_{\parallel} + R_{\perp})/2 =$	164.0	164.5	176.5	208.2	222.7	235.3	263.3	280.4	326.9	603.0	836.5	836.5	
p(Stokes) =	18.58	37.13	64.89	91.71	94.99	96.93	97.92	97.71	96.53	70.96	50.54	50.54	
ζ (Stokes) =	-11.60	-3.97	-0.74	1.07	1.39	1.40	0.88	1.52	1.52	3.86	4.07	4.07	
Theoretical													
$\theta_t =$	13.53	20.00	26.09	31.61	32.63	33.61	34.56	35.47	36.34	40.01	41.37	41.37	
$R_{\parallel} =$	2.93	2.19	1.21	0.24	0.11	0.02	0.00	0.06	0.24	4.42	10.87	10.87	
$R_{\perp} =$	4.16	5.13	6.91	10.17	11.10	12.16	13.37	14.74	16.31	28.30	38.21	38.21	
$R_{\parallel} + R_{\perp} =$	7.08	7.32	8.12	10.41	11.21	12.19	13.37	14.80	16.54	32.71	49.09	49.09	
$R_{\perp}/(R_{\parallel} + R_{\perp}) =$	58.69	70.13	85.15	97.69	99.02	99.80	99.99	99.57	98.57	86.50	77.85	77.85	Index of Refraction
$R_{\parallel}/(R_{\parallel} + R_{\perp}) =$	41.31	29.87	14.85	2.31	0.98	0.20	0.01	0.43	1.43	13.50	22.15	22.15	1.461575
Difference =	0.16	1.74	2.72	1.87	1.58	1.40	1.05	0.79	0.37	1.34	2.83	2.83	$\Sigma (\text{Difference})^2$
(Difference) ² =	0.02	3.03	7.39	3.49	2.51	1.95	1.11	0.62	0.14	1.80	8.04	8.04	30.08945

Figure 6. Excel data table for experiment using a glass slide (index of refraction = 1.46).

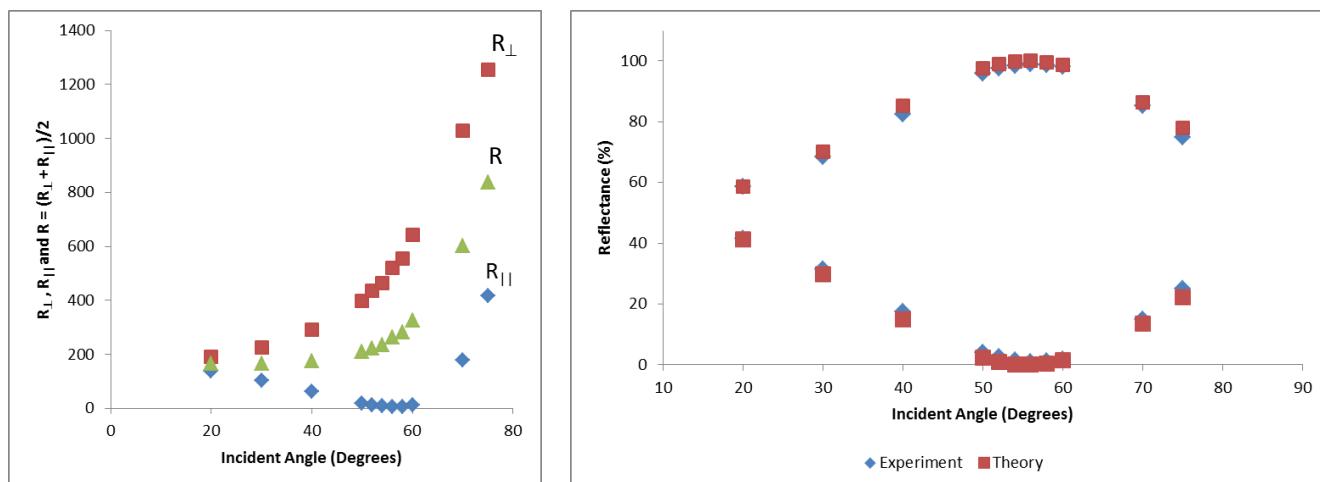


Figure 7. Left shows the measurements of R_{\perp} (red \blacksquare), R_{\parallel} (blue \blacklozenge) and $R = (R_{\perp} + R_{\parallel})/2$ (green \blacktriangle). Right is a graph of the partial polarization (top is R_{\perp} (%) and bottom is R_{\parallel} (%)).

4. Experimental Analysis Notes

Once students have obtained and entered their data in an Excel spreadsheet, they are then asked to analyze the data. Below are some of the results that students can obtain and assistance may be necessary depending on the student's level of experience.

A) From the plot of R_{\parallel} the most obvious feature is the decrease in intensity of the light at the Brewster angle. The fact that the data is plotted using Excel normally precludes a cubic spline fit to get a more precise value, although there are second party programs that can be added to Excel to incorporate cubic splines⁵. However, observing the general behavior and noting that the Brewster angle is very close to the accepted value for the dielectric material being used is sufficient for this experiment and the qualitative behavior of R_{\perp} , R_{\parallel} , and $R = (R_{\perp} + R_{\parallel})/2$ should be compared with theory.

B) The partial polarization shown in Figure 7 (right) should be compared with the expected theoretical result. This is shown in the figure where the blue diamonds are the experimental results and the red squares the expected theoretical results. Since the index of refraction is typically wavelength dependent, using a light source that is not monochromatic should be a consideration when selecting a dielectric material. In this experiment a glass microscope slide was used because its dispersion is very small over optical wavelengths. When choosing the dielectric material an excellent resource for the index of refraction as a function of wavelength for different materials is the website refractiveindex.info⁶ (which contains other information as well as references).

Students are asked to fit the data using Excel's solver routine and an outline of the method follows. First, the difference between the experimental and theoretical values are computed and then squared [the line $(\text{Difference})^2$ at the end of the data table]. These are then summed and it is the sum of the squared differences that is minimized to achieve the best fit by the Solver routine that is found under the Data tab. If it is not there the student will have to add it in by clicking File \rightarrow Options \rightarrow Add-in \rightarrow [Manage: Excel Add-in] \rightarrow Go and selecting Solver Add-in \rightarrow Go. Then click the Data tab again and the Solver routine should be at the end under Data Analysis in the Analysis pane. Clicking on Solver brings up the Solver Parameters dialog box shown in Figure 8.

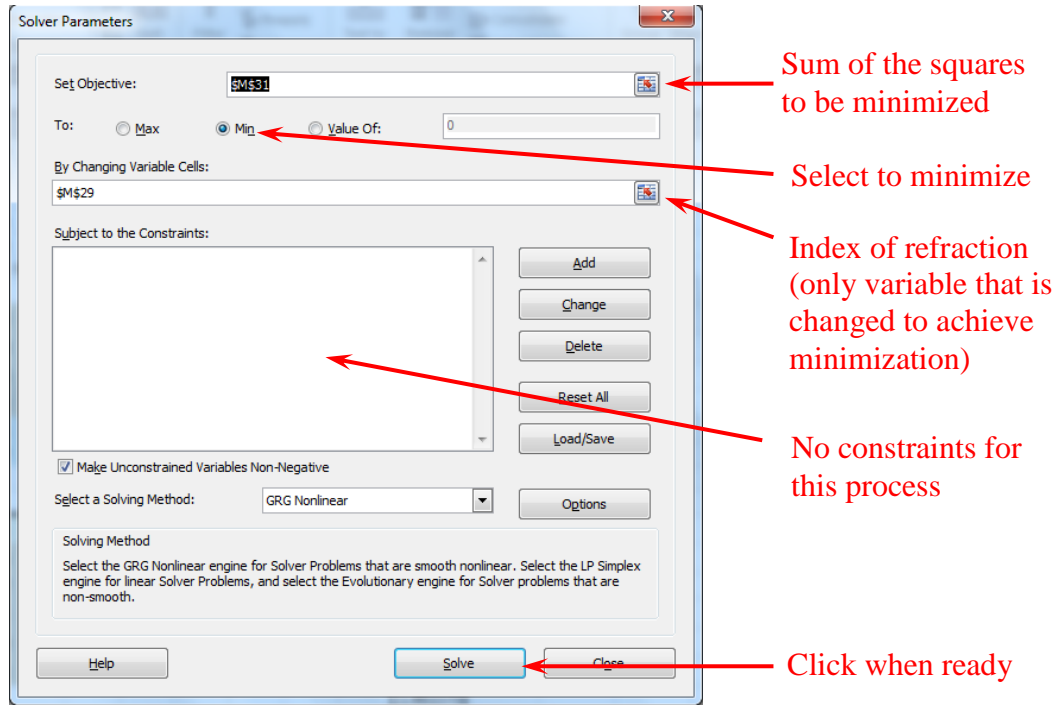


Figure 8. Solver parameter dialog box. Enter the appropriate cell locations in the Excel file.

Once the student has entered the appropriate cells from the Excel file into the Solver parameter dialog box, clicking Solve will minimize the sum of the squared differences and yield a value of the index of refraction. In this experiment the substrate is fused silica that has an index of refraction in the visible of 1.46 and the best nonlinear fit value is 1.462.

C) The experiment can be performed with polarizer measurements of 0° and 90° but it's just as easy to obtain measurements at 45° and 135° and requires only a little more time. However, what is gained is the polarization measured using the Stokes parameters³. The normalized Stokes parameters q and u are easily obtained using the measured values and the equations

$$q = \frac{Q}{I} = \frac{I_{0^\circ} - I_{90^\circ}}{I_{0^\circ} + I_{90^\circ}} \quad (5)$$

$$u = \frac{U}{I} = \frac{I_{45^\circ} - I_{135^\circ}}{I_{0^\circ} + I_{90^\circ}} \quad (6)$$

with the degree of polarization given by $p = \sqrt{q^2 + u^2}$ and the position angle with respect to the fiducial setting given by $\tan(2\zeta) = u / q$. The degree of polarization and position angle are shown in the data table of Figure 6 as $p(\text{Stokes})$ and $\zeta(\text{Stokes})$. The student should notice not only the trend in the degree of polarization (increasing, reaching maximum at the Brewster angle, then decreasing) but also the position angle (quickly approaching polarization perpendicular to the plane-of-incidence, reaching its maximum value at the Brewster angle, then slightly increasing). In discussing partial polarization, usually a qualitative description of the polarization position angle is given, noting that the vector is perpendicular to the plane-of-incidence at the Brewster angle, but here it is actually "observed" and shown to be fairly perpendicular over a wide range of incidence angles.

Also the student should note that the difference between the perpendicular and parallel partial polarizations $[R_{\perp}(\%) - R_{\parallel}(\%)]$ is (to within experimental accuracy) the degree of polarization determined by using the Stokes parameters. A qualitative explanation is that the Stokes parameters measure the degree of polarization and not the tendency to be polarized perpendicular or parallel to the plane-of-incidence; it is the actual amount of polarization that is observed. However, in terms of partial polarization, the vectors perpendicular to the plane-of-incidence do not completely cancel all of the vectors that are parallel to the plane-of-incidence, hence there are “left over” polarization vectors, which is the polarization of the beam.

D) There are sources of error and uncertainty in this experiment. While the focus is on the student experience over the entire exercise, they should nevertheless note some of the possible sources of uncertainty and suggest improvements to achieve more accurate results. Items that should be discussed include: background light, scattering, monochromatic light, detector polarization, reflections, and partial polarization of the source. A measurement that should be made is of the light source itself without the polarizer and accurately rotating the detector head through the same four position angles to determine the degree of polarization using the Stokes parameters. This is shown in the data table in Figure 6 at the top right. The measurements indicate that the polarization is on the order of 0.6% and, since the source and detector head are not being assessed separately, the polarization is a combination of the source and detector head. However, the result shows that the amount of polarization is small, but nonetheless, still present.

5. Conclusion

In this paper we have presented a laboratory exercise on partial polarization. From the design, construction, and execution of the exercise, the students have a hands-on approach to a more realistic optics laboratory experience that challenges them to go beyond the formulistic, mechanical approach of similar turn-key purchased exercises. The explanation of the partial polarization measurements obtained and the introduction of the Stokes parameters helps students to visualize the physics and should have a better appreciation of partial polarization.

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