Barrier Lyapunov function-based adaptive control for uncertain linear motor systems

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ABSTRACT

In this work, the tracking problem for permanent magnetic synchronous linear motor systems is studied. We develop a novel barrier Lyapunov function-based adaptive control scheme for linear motor systems with system constraints. The filtering-error is constrained by using a new type BLF so as to simplify design, which is different from the existing results. The time-varying boundary layer technique is introduced to reduce the difficulty of choosing the barrier parameter. Also, a neural network is adopted for dealing with nonparametric uncertainties. In the end, a simulation example is presented to demonstrate the effectiveness of our barrier adaptive control algorithm against traditional barrier-free adaptive control algorithm.

Keywords: Adaptive control, linear motor systems, barrier Lyapunov functions

1. INTRODUCTION

The rapid progress of adaptive control technology has been made during the past decades[1-3]. A great number of adaptive control algorithms have been proposed for parametric uncertain systems. However, in many cases, the unknown nonlinearities cannot be linearly parameterizable. There often are two strategies to handle these uncertainties. Firstly, for a certain nonparametric uncertainty, robust control may be used to develop a feedback compensation according to the upper bound of uncertainty[4]. Secondly, with neural networks[5] and fuzzy logic systems[6] constructed, adaptive control based on neural networks or fuzzy logic systems are effective in dealing with nonparametric uncertainties.

In real situations, the controlled objects are inevitable to meet with various system constraints, which has a large or small impact on the stability and performance of systems. In the past decade, the research on barrier adaptive control approaches has attracted increasing interests, with various barrier Lyapunov functions constructed to solve the trajectory-tracking problems under system constraints[7,8]. In the controller design of barrier Lyapunov function-based (BLF-based) adaptive control, the constrained objects are the system output, the system state, or the output tracking error. The BLF-based adaptive control strategy is effective to improve the robustness and the safety of systems[9].

Permanent magnetic synchronous linear motors (PMSLMs) possess some distinctive characteristics[10], that is, they have simpler structure, bigger thrust force and higher precision than common rotational motors. Due to these merits, PMSLMs have been widely applied in numerous cases. The adaptive control for PMSLM systems has been studied for long[11,12]. So far, some related results on the adaptive control for permanent magnetic with state/output constraints[13] have been proposed, focusing on backstepping technique together with barrier Lyapunov functions. There are still some problems that need to be further studied. For one thing, it is not very convenient to design the virtual control laws according to backstepping technique. For another, while applying these BLF-based adaptive control algorithms, how to set the value of the barrier parameter in barrier Lyapunov function is not an easy job, since the closed-loop system cannot work properly whether the barrier parameter is chosen too small or too large.

To solve these above-mentioned problems, in this work, we develop a BLF-based adaptive control scheme. Through constructing a new type BLF, the filtering-error is constrained. The time-varying boundary layer technique is introduced to reduce the difficulty of choosing the barrier parameter. Also, a neural network is adopted for approximating nonparametric uncertainties.

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2. PROBLEM FORMULATION

The system dynamic model of a class of PMSLMs may be described by

$$\begin{cases} \dot{s}(t) = v(t) \\ M\dot{v}(t) = F_m - D_f(t) - D_r(t) - D_u(t) \end{cases} \tag{1}$$

where t is time, s is the displacement, v is the speed, M denotes the total mass of mover and load, and F_m is the driving force of motor. $D_f(t)$, $D_r(t)$ and $D_u(t)$ represent unknown friction, force ripple, and external disturbance, respectively.

Let $x_1=s$, $x_2=v$, $u=F_m$. Then, the system model of linear motor in the kth iteration cycle may be obtained as

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = au(t) + a(D_f(t) + D_r(t) + D_u(t)) \\ y_1(t) = x_1(t) \end{cases} \tag{2}$$

where $a = \frac{1}{M}$. The reference trajectory is $y_d(t)$. Our control objective is to design a proper controller u, so that $y(t)$ can accurately track $y_d(t)$, as well as to constrain the system states during system operation. In the following of this paper, the arguments of functions are often omitted for brevity.

3. CONTROLLER DESIGN

Let us define $e_{1,k} = x_1 - y_d$, $e_{2,k} = x_2 - \dot{y}_d$ and $s = ce_1 + e_2$, where $c > 0$ is a design parameter. From equation (2), we have

$$\begin{cases} \dot{e}_1 = e_2, \\ \dot{e}_2 = au(t) + a(D_f(t) + D_r(t) + D_u(t)) - \ddot{y}_d \end{cases} \tag{3}$$

Let us construct a time-varying boundary layer as $s_\phi = s - \phi \mathrm{sat}(\frac{s}{\phi})$, $\phi = |s(0)| \mathrm{e}^{-\lambda t}$, where $\lambda > 0$ and

$$\mathrm{sat}(\hat{a}) = \begin{cases} \hat{a} & \text{if } |\hat{a}|<1 \\ \mathrm{sgn}(\hat{a}) & \text{if } |\hat{a}| \geq 1 \end{cases}$$

Let us choose a candidate barrier Lyapunov function

$$V = \frac{s_\phi^2}{2a(b_s^2 - s_\phi^2)}, \tag{4}$$

with $b_s > 0$. Then, we can get $\dot{V}$ as

$$\dot{V} = \sigma s_\phi [a^{-1} c e_2 + u + D_f + D_r + D_u - a^{-1} \ddot{y}_d], \tag{5}$$

with $\sigma = \frac{b_s^2}{(b_s - s_\phi^2)^2}$. To compensate for the uncertainties, an RBF neural network is introduced to approximate $D_f + D_r + D_u$ as follows:

$$D_f + D_r + D_u = w^T h(z) + \varepsilon \tag{6}$$

where ε is the bounded approximation error,

$$h(z)=[h_1(z), \quad h_2(z), \quad ..., \quad h_n(z)]^T,$$

$$h_j(z)=\exp(-\frac{\| z-\boldsymbol{c}_j \|}{2b_j^2}) \quad .$$

Here,

$$z=[x_1, \quad x_{2,} \quad e_1, \quad e_2]^T, \; b_j>0, \quad j=1,2,...,m,$$

$$\boldsymbol{c}_j=[c_{j1}, \; c_{j2}, \; ..., c_{jn}]^T.$$

Substituding equation (6) into equation (5), we have

$$\dot{V}=\sigma s[a^{-1}ce_2+u+w^T h(z)+\varepsilon-a^{-1}\ddot{y}_d]=\sigma s[\theta^T\varphi+w^T h(z)+\varepsilon+u], \tag{7}$$

where $\theta=[a^{-1}c,-a^{-1}]^T$, $\varphi=[e_2,\ddot{y}_d]^T$.

Then, we design the following control law and adaptive laws as

$$u=-\mu(b_s^2-s_\phi^2)s_\phi-\beta\sigma s_\phi-\hat{\boldsymbol{\theta}}^T\boldsymbol{\varphi}-\hat{w}^T h(z), \tag{8}$$

$$\dot{\hat{\boldsymbol{\theta}}}=\sigma\gamma_1 s_\phi\varphi-k\gamma_1\hat{\theta}, \tag{9}$$

$$\dot{\hat{w}}=\sigma\gamma_2 s_\phi h(z)-l\gamma_2\hat{w}, \tag{10}$$

where $\mu>0,\beta>0,\gamma_1>0,\gamma_2>0,k>0,l>0$.

Remark 1. In traditional BLF-based adaptive control algorithms, how to set the barrier parameter b_e in $V=\ln\frac{b_e^2}{b_e^2-e^2}$ is not easy. b_e should be set properly so that $b_e^2-e^2(0)>0$. However, if the value of b_e is set too big, the effectiveness of BLF-adaptive control algorithms is fewer. In our design, $s_\phi(t)=0$ holds, so the above-mentioned difficulty is overcome.

4. CONVERGENCE ANALYSIS

Theorem 1: For the closed-loop system consisting of system model (1), control law (8) and adaptive laws (9)-(0). All signals are bounded, $s_\phi<b_s$ is guaranteed and

$$\lim_{t\to\infty}|s(t)|\le b_s\sqrt{\frac{2a\lambda_2}{\lambda_1}} \,. \tag{11}$$

Proof:

Substituting equation (8) into equation (7), we have

$$\dot{V}=\sigma s_\phi[\tilde{\theta}^T\varphi+\tilde{w}^T h(z)+\varepsilon+u] \tag{12}$$

Then, we define another Lyapunov function

$$L=V+\frac{\tilde{\theta}^T\tilde{\theta}}{2\gamma_1}+\frac{\tilde{w}^T\tilde{w}}{2\gamma_2} \,. \tag{13}$$

By using equations (9) and (10), we can take the time derivative of L as

$$\begin{aligned}\dot{L} &= -\frac{\mu s_\phi^2}{b_s^2 - s_\phi^2} + \sigma s_\phi[\tilde{\theta}^T\varphi + \tilde{w}^T h(z) + \varepsilon] + \frac{\tilde{w}^T\tilde{w}}{2\gamma_2} + \frac{1}{\gamma_1}\tilde{\theta}^T(-(\sigma\gamma_1 s_\phi \varphi - k\gamma_1\hat{\theta})) \\ &+ \frac{1}{\gamma_2}\tilde{w}^T(-(\sigma\gamma_2 s_\phi h(z) - l\gamma_2\hat{w})) - \beta\sigma^2 s_\phi^2 \\ &= -\frac{\mu s_\phi^2}{b_s^2 - s_\phi^2} + \sigma s_\phi \varepsilon + \tilde{\theta}^T k(\theta - \tilde{\theta}) + \tilde{w}^T l(w - \tilde{w}) - \beta\sigma^2 s_\phi^2\end{aligned} \tag{14}$$

Note that

$$\begin{aligned}&\tilde{\theta}^T k(\theta - \tilde{\theta}) \\ &= -k\tilde{\theta}^T\tilde{\theta} + k\tilde{\theta}^T\theta - k\theta^T\theta + k\theta^T\theta \\ &= -\frac{k}{2}\tilde{\theta}^T\tilde{\theta} - \frac{k}{2}\tilde{\theta}^T\tilde{\theta} + k\tilde{\theta}^T\theta - \frac{1}{2}k\theta^T\theta + \frac{1}{2}k\theta^T\theta \\ &= -\frac{k}{2}\tilde{\theta}^T\tilde{\theta} - \frac{k}{2}(\tilde{\theta} - \theta)^T(\tilde{\theta} - \theta) + \frac{1}{2}k\theta^T\theta\end{aligned} \tag{15}$$

and

$$\begin{aligned}&\tilde{w}^T l(w - \tilde{w}) \\ &= -l\tilde{w}^T\tilde{w} + l\tilde{w}^T w - lw^T w + lw^T w \\ &= -\frac{l}{2}\tilde{w}^T\tilde{w} - \frac{l}{2}\tilde{w}^T\tilde{w} + l\tilde{w}^T w - \frac{1}{2}lw^T w + \frac{1}{2}lw^T w \\ &= -\frac{l}{2}\tilde{w}^T\tilde{w} - \frac{l}{2}(\tilde{w} - w)^T(\tilde{w} - w) + \frac{1}{2}lw^T w.\end{aligned} \tag{16}$$

Substituting equations (15) and (16) into equation (14), we have

$$\begin{aligned}\dot{L} &\le -\frac{\mu s_\phi^2}{b_s^2 - s_\phi^2} + \sigma s_\phi \varepsilon - \frac{k}{2}\tilde{\theta}^T\tilde{\theta} + \frac{1}{2}k\theta^T\theta - \frac{l}{2}\tilde{w}^T\tilde{w} + \frac{1}{2}lw^T w - \beta\sigma^2 s_\phi^2 \\ &\le -\frac{\mu s_\phi^2}{2a(b_s^2 - s_\phi^2)} + \sigma s_\phi \varepsilon - \frac{k\gamma_1}{2\gamma_1}\tilde{\theta}^T\tilde{\theta} - \frac{l\gamma_2}{2\gamma_2}\tilde{w}^T\tilde{w} + \frac{1}{2}k\theta^T\theta + \frac{1}{2}lw^T w - \beta\sigma^2 s_\phi^2 \\ &\le -\frac{\mu s_\phi^2}{2a(b_s^2 - s_\phi^2)} + \sigma s_\phi \varepsilon - \frac{k\gamma_1}{2\gamma_1}\tilde{\theta}^T\tilde{\theta} - \frac{l\gamma_2}{2\gamma_2}\tilde{w}^T\tilde{w} + \frac{1}{2}k\theta^T\theta + \frac{1}{2}lw^T w - \beta\sigma^2 s_\phi^2\end{aligned} \tag{17}$$

By using Young's inequality, we have

$$\sigma s_\phi \varepsilon \le \frac{1}{4\beta}\varepsilon^2 + \beta\sigma^2 s_\phi^2 \tag{18}$$

Combining equation (17) with equation (18), we obtain

$$\begin{aligned}\dot{L} &\le -\frac{2a\mu s_\phi^2}{2a(b_s^2 - s_\phi^2)} - \frac{k\gamma_1}{2\gamma_1}\tilde{\theta}^T\tilde{\theta} - \frac{l\gamma_2}{2\gamma_2}\tilde{w}^T\tilde{w} + \frac{1}{2}k\theta^T\theta + \frac{1}{2}lw^T w + \frac{1}{4\beta}\varepsilon^2 \\ &\le -\lambda_1 L + \lambda_2,\end{aligned} \tag{19}$$

in which, $\lambda_1=\min(2a\mu,k\gamma_1,l\gamma_2),\lambda_2=\sup(\frac{1}{2}k\theta^T\theta+\frac{1}{2}lw^Tw+\frac{1}{4\beta}\varepsilon^2)$. By Lemma B.5 in Reference[5], from equation (19), we have

$$L(t)\ \le \mathrm{e}^{-\lambda_1 t}L(0)+\lambda_2\int_0^t \mathrm{e}^{-\lambda_1(t-\tau)}d\tau \le \mathrm{e}^{-\lambda_1 t}[L(0)-\frac{\lambda_2}{\lambda_1}]+\frac{\lambda_2}{\lambda_1} \tag{20}$$

Since $L(0)$ is bounded, from equation (20), we can see that $L(t)$ is bounded for $t>0$. Then, due to the fact that $V\le L$ holds, i.e.,

$$\frac{s_\phi^2(t)}{2a(b_s^2-s_\phi^2(t))}<\infty,\ \forall t>0.$$

Therefore, $|s_\phi(t)|<b_s$ holds for $t>0$. Further, from equation (20), we can conclude

$$s_\phi^2(t)\le 2ab_s^2[\mathrm{e}^{-\lambda_1 t}(L(0)-\frac{\lambda_2}{\lambda_1})+\frac{\lambda_2}{\lambda_1}]$$

and

$$\lim_{t\to\infty}|s_\phi(t)|\le b_s\sqrt{\frac{2a\lambda_2}{\lambda_1}}.$$

By this and the definition of s_ϕ, we obtain

$$\lim_{t\to\infty}|s(t)|\le b_s\sqrt{\frac{2a\lambda_2}{\lambda_1}}.$$

In our control scheme, the filtering error is constrained. Comparing to the backstepping based adaptive control with error constraints, our control algorithm is easier for implementation.

5. NUMERICAL SIMULATION

Considering the system (1) whose parameters are set as follows: $M=5\mathrm{kg}\cdot\mathrm{m}^2$, B=0.27, $D_f+D_r+D_u=2+0.5\sin(2\pi t)+(0.25+0.2x_1x_2)\mathrm{sgn}(x_2)+3\sin(3\pi t)$. We take the adaptive ILC algorithms (18)-(20) for simulation, with $\mu=5,\beta=5,\gamma_1>15,\gamma_2>15,k>0.1,l>0.1,b_s=0.4$. The reference signal is $x_{1,d}=\cos(0.4\pi t)$ and $x_{2,d}=-0.4\pi\sin(0.4\pi t)$.The position tracking profile and velocity tracking profile are shown in Figures 1 and 2, respectively. The profiles tracking error are given in Figures 3 and 4. Figure 5 show the value of control input. We can see the control input signal is smooth is smooth along the time axis. The profile of $|s_\phi(t)|$ is provided in Figure 6, from which we can see that $|s_\phi(t)|<b_s$ holds for $t\ge 0$. For comparison, we take a barrier-free adaptive control algorithm for simulation as follows:

$$u=\beta s_\phi-\hat{\boldsymbol{\theta}}^T\boldsymbol{\varphi}-\hat{w}^Th(z)\,, \tag{8}$$

$$\dot{\hat{\boldsymbol{\theta}}}=\gamma_1 s_\phi\varphi-k\gamma_1\hat{\theta}\,, \tag{9}$$

$$\dot{\hat{w}}=\gamma_2 s_\phi h(z)-l\gamma_2\hat{w}\,, \tag{10}$$

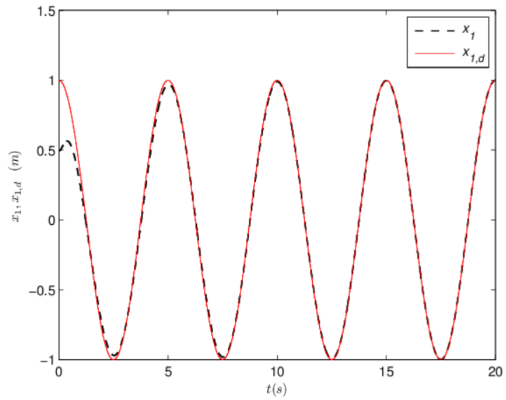

Figure 1. Position trajectory x_1 (barrier ILC).

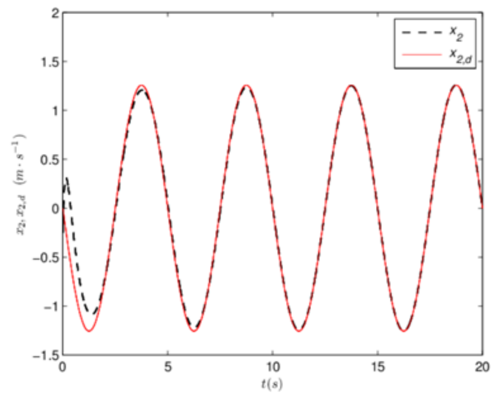

Figure 2. Velocity trajectory x_2 (barrier ILC).

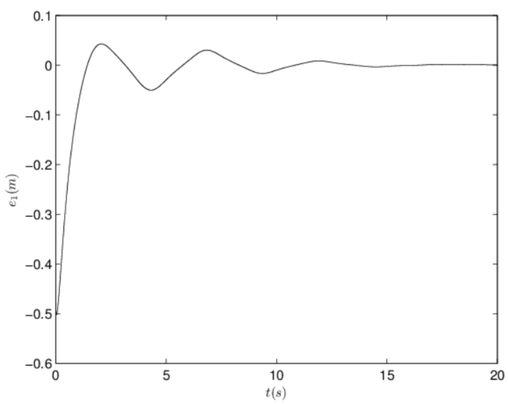

Figure 3. position error e_1 (barrier ILC).

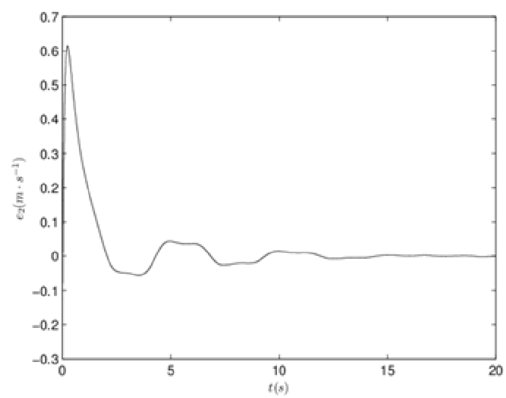

Figure 4. velocity error e_2 (barrier ILC).

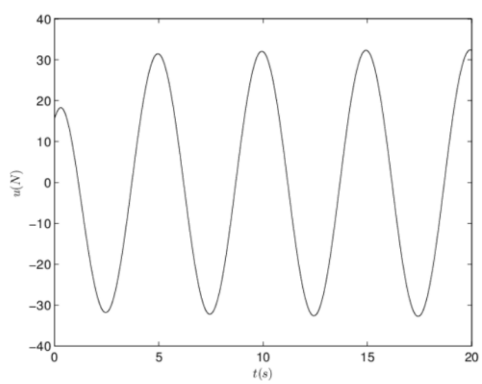

Figure 5. Control input (barrier ILC).

The profile of $|s_\phi(t)|$ in barrier-free adaptive control algorithm is illustrated in figure 7, from which we can see that $|s_\phi(t)| < b_s$ is violated. Comparing Figure 6 with Figure 7, the effectiveness of our proposed adaptive control is demonstrated.

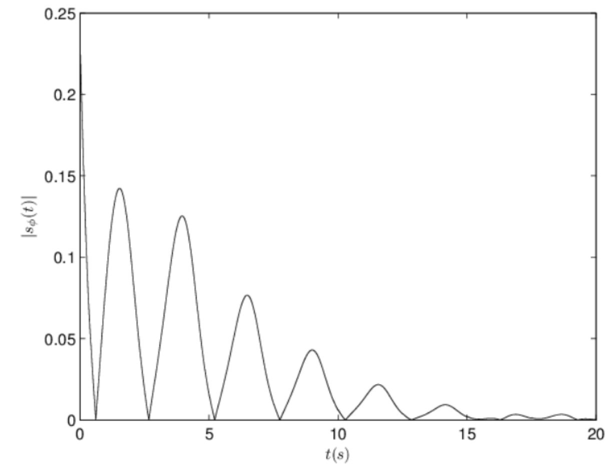

Figure 6. $|s_\phi(t)|$ (barrier ILC).

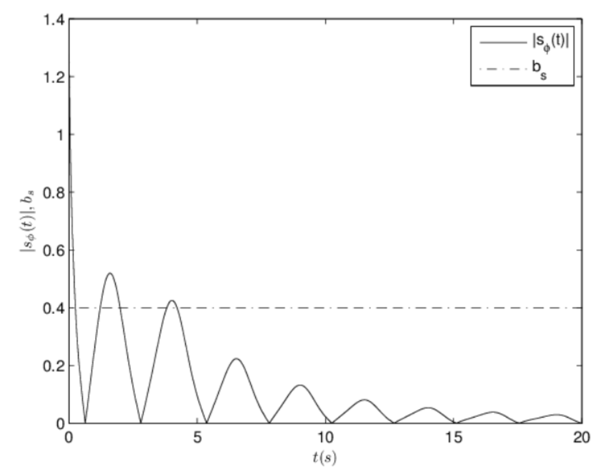

Figure 7. $|s_\phi(t)|$ (barrier-free ILC).

6. CONCLUSION

This paper discusses the trajectory tracking problem for linear motor systems with system constraints.

A novel barrier Lyapunov function is constructed to develop the adaptive control scheme, such that the filtering-error during each iteration is constrained in the presented range. The time-varying boundary layer technique is used to simplify the complexity of controller design, with neural network used to approximate nonparametric uncertainties.

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