Computer - generated hologram optical elements in basic optics course

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ABSTRACT

The principle of operating and manufacture of kinoform's elements is described. As examples the first approximations of axisymmetrical lens kinoform: two-stage phase plate (Wood's plate), three- and four - stage phase plate are considered.

1. INTRODUCTION

Rapid growth of optics in last decades connected with creation of coherent light sources is embodied in content and education procedure of basic optics course.The high coherence of laser radiation presents a possibility significantly to enrich lectures and practice by new demonstrations and laboratories.

Intensive growth of modern optical technique - optics of kinoforms - has opened up new perspectives[1-5]. At the heart of the kinoform is the possibility of complex wave amplitude control by variation of sample thickness or refraction index. Kinoform is intermediate optical system between diffraction and refraction one. In kinoforms as in diffraction systems the optical pathlengths of all beams connecting the object and his image are not constant and change passing from one zone to another by jumps equal to one wavelength. At the same time the kinoforms look like a refractive systems because the optical pathlengths of beams level off by variation of sample thickness[6].

The kinoforms offer few advantages over the traditional optical elements - lenses, prisms, mirrors. There are a small weight and dimensions. This is because the forming of phase front in kinoforms occurs on the thicknesses equal approximately to one wavelength. As a rule, kinoforms are destined for operating in monochromatic light, that is why these elements are widely used in coherent optics.

Some simple kinoforms may be used for creation new demonstrations and laboratories.

2. THE RECORDING AND THE RECONSTRUCTION OF WAVE FRONT USING THIN HOLOGRAMS

Let us assume that in plane z=0 of Cartesian coordinate x,y,z (Fig.1) the interference picture, formed by sample (s)

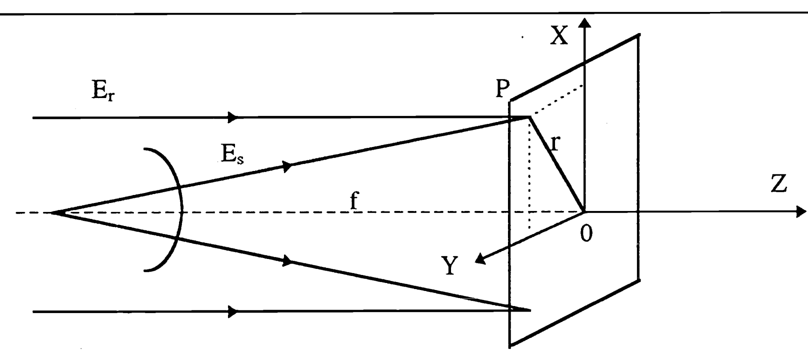

Fig.1. The interference of plane reference (E_r) and spheric sample (E_s) waves.

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and reference (r) monochromatic waves with the complex amplitudes:

$$E_s = A_s\, exp(i\varphi_s) \quad , \quad E_r = A_r\, exp(i\varphi_r) \quad , \tag{1}$$

is recorded. Here $A_{s,r}$ and $\varphi_{s,r}$ denote the amplitudes and phases of s and r waves respectively.
The intensity distribution in z=0 plane is given by

$$I \propto \left|E_s + E_r\right|^2 = A_s^2 + A_r^2 + 2A_s A_r Cos\psi \quad , \tag{2}$$

where $\psi = \varphi_s - \varphi_r$.
After recording this distribution with photographic materials and appropriate chemical treatment, we shall obtain the hologram. The transmittance of the hologram can be written as:

$$(x,y) = \rho(x,y)\, exp\left[i\delta(x,y)\right] \quad , \tag{3}$$

where $\rho(x,y)$ - amplitade of transmittance ($0 \le \rho \le 1$) and $\delta(x,y)$ - the phase delay ($|\delta(x,y)| \le 2\pi$).

The transmittance of hologram is a function of the intensity Eq.(2). So, the transmittance will be a regular function of the phase difference: $(\psi) = \tau(\psi + 2\pi)$ and it can be written as a series expansion:

$$(\psi) = \sum_{-\infty}^{+\infty} \tilde{N}_m\, exp(im\psi), \tag{4}$$

where the coefficient

$$_m = \frac{1}{2\pi}\int_0^{2\pi} \tau(\psi)\, exp(-im\psi)\, d\psi \tag{5}$$

is the amplitude of m-th component of the transmittance function.

If the hologram is illuminated by the wave $E_v = A_v\, exp(i\varphi_v)$, the complex wave amplitude just beyond the hologram can be written as:

$$E_d = E_v \tau(\psi) = \sum_{m=-\infty}^{+\infty} A_v C_m\, exp\left[i(m\psi + \varphi_v)\right] = \sum_{m=-\infty}^{+\infty} E_m \quad . \tag{6}$$

Here E_m is the complex wave amplitude in the m-th diffraction order.

The diffraction efficiency in the m-th order is determined as:

$$\mu_m = \frac{\left|E_m\right|^2}{\left|E_v\right|^2} = \left|C_m\right|^2 \quad . \tag{7}$$

If the reconstractive wave is identical to reference one,

$$E_v = A_v\, exp(i\varphi_r) \quad , \tag{8}$$

then the sample wave will be formed in the first diffraction order with amplitude $A_v C_1$.
Really,

$$E_1 = A_v\, exp(i\varphi_r)\, exp\left[i(\varphi_s - \varphi_r)\right] = A_v C_1\, exp(i\varphi_s) \quad . \tag{9}$$

3. MULTILEVEL PHASE HOLOGRAM. KINOFORMS.

Now we determine the transmittance $\tau(\psi)$ of hologram, which has a diffraction efficiency $\mu_1=1$. Obviously that holograms will be pure phase plates with transmittance function:

$$(\psi) = exp\left[i\delta(\psi)\right] \quad . \tag{10}$$

Those plates are said to be kinoform[6].

The diffraction efficiency μ_1 follows from Eqs.(5) and (7) can be written as:

$$\mu_1 = \left| \frac{1}{2\pi} \int_0^{2\pi} exp[i\delta(\psi)]\, exp(-i\psi) d\psi \right|^2 \quad . \tag{11}$$

$\mu_1 = 1$, if:

$$\delta(\psi) - \psi = 0 \quad . \tag{12}$$

Since $|\delta(\psi)| \le 2\pi$, it is evident that the part of phase difference multiple of 2π can be discard. This part is not essential in forming the interference picture Eq.(2). Thus:

$$\delta(\psi) = \psi - 2\pi \cdot ent\left(\frac{\psi}{2\pi}\right) \quad . \tag{13}$$

The relief-phase holograms are the most commonly encountered among phase holograms. These holograms introduce the phase change $\delta(\psi)$, which is connected with height of relief $h(\psi)$. For example, for transparent hologram:

$$\delta(\psi) = \frac{2\pi}{\lambda_0} h(\psi) \cdot (n-1) \quad , \tag{14}$$

where n - the refractive index, λ_0 - the wavelength.

Thus, the height of relief $h(\psi)$ can be written as (see Fig.2a):

$$h(\psi) = \frac{\lambda_0}{2\pi(n-1)} \left[\psi - 2\pi \cdot ent\left(\frac{\psi}{2\pi}\right) \right] . \tag{15}$$

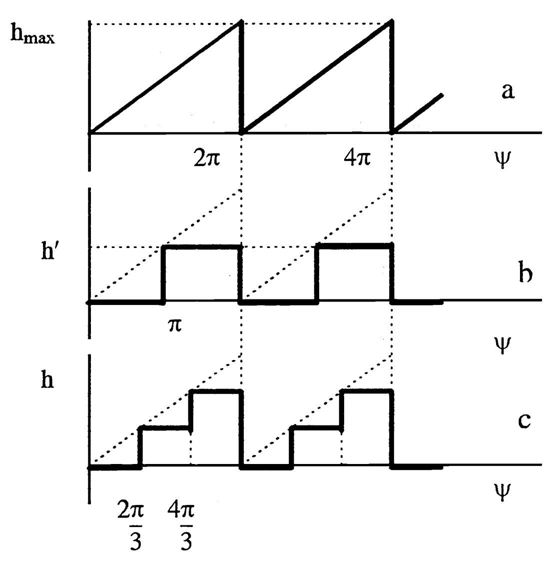

Fig.2. Relief height as a function of ψ: a - for kinoform, b - for two-level phase plate with τ given by Eq.(17), c - three-level phase plate with τ given by Eq.(23).

The maximum of height is given by:

$$h_{max} = \lambda_0 / (n-1) \tag{16}$$

A first approximation of kinoform is a binary phase hologram, which has two levels of phase change (two-level hologram). The maximum of diffraction efficiency in the first order can be reached at:

$$\tau(\psi) = \begin{cases} 1, & 2\pi l < \psi \le \pi + 2\pi l \\ exp(i\pi), & \pi + 2\pi l < \psi \le 2\pi(l+1) \end{cases}$$
$$l = 0, \pm 1, \pm 2, \dots \tag{17}$$

The relation between h and ψ is presented in Fig.2b, where:

$$h' = \lambda_0 / 2(n-1) \quad . \tag{18}$$

Substitution Eq.(17) in Eq.(5) and (7) gives diffraction efficiency of two-level hologram:

$$\mu_0 = 0, \quad \mu_m = \left(\frac{1-\cos\pi m}{\pi m}\right)^2, \quad m = \pm 1, \pm 2 \ldots \tag{19}$$

From Eq.(19) it is seen that the zeroth and even diffraction orders are absent; the intensity of ±m-th order is equal and the difraction efficiency decreases as $1/m^2$. The diffraction efficiency of first order is:

$$\mu_{\pm 1} = \frac{4}{\pi^2} = 0.405 \quad . \tag{20}$$

The widely known example of two-level phase hologram is Wood's plate. The geometrical place of points with uniform phase difference will be concentric rings (zones). The outer radius r_k coincides with the radius of Fresnel's zone for small k:

$$r_k = \sqrt{kf\lambda} \quad , \tag{21}$$

where f - "focus length".

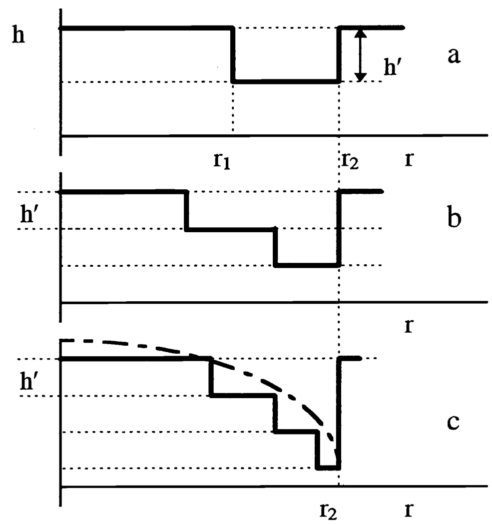

Fig.3 Thiknesses of N-levels phase plates as function of radius. N=2 (a), 3 (b), 4 (c). Dashed curve - kinoform of spherical lens.

Unlike the amplitude zone plate, all zones are transparent, and their thicknesses differ by the value h' (see Eq.(18), Fig.3a). In this case the phase difference from neighbouring zones has additional changes π. The waves E_m Eq.(6) are converging and diverging spherical waves. Wood's plate, in fact, is a first approximation to kinoform of spherical lens. However, unlike kinoform this plate has a few focus lengths:

$$f_m = f/m \tag{22}$$

where m - are the odd integers.

If a Wood's plate is illuminated by a plane monochromatic wavefield as depicted in Fig.4, the transmitted wavefield will be consisted of spherical waves. Their intensities may be determined from Eq.(19). We may say that Wood's plate plays the role of negative and positive lens at a time.

The operation of Wood's plate is readily illustrated by the vector diagram. Let the plate is absent. The amplitudes of the vibrations A_1 and A_2 from the first and second Fresnel's zones for point P_1 (Fig.4) are shown in Fig.5a. The overall amplitude from all zones in this case, as is well-known, approximately equal to a half of amplitude A_1.

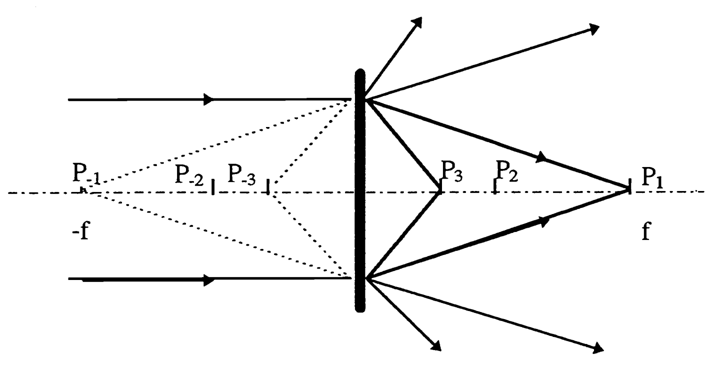

Fig.4 Diffraction of plane monochromatic wave on Wood's plate.

Wood's plate introduces the additional phase difference between neighboring zones equal to π. So, the vectors A_2, A_4, ... are rotated through π (Fig.5b). As a result the amplitudes from the all zones increase considerably

As the screen is moved closer to plate at the distance f/2 (P_2, Fig.4), the radii of new Fresnel's zones will be decreased. Then two new Fresnel's zones will be placed within the one step of phase plate. The waves from these zones are always π out of phase and attenuate each other in point P_2 considerably.

Similar reasoning for points P_4, P_6, ... are spaced at f/4, f/6, ... distancies from the plate shows that the intensities are equal to zero. By this means the lack of even-numbered order can be explained (see Eq.(19)).

For point P_3 is spaced at f/3 distance from plate (Fig.4), there are three Fresnel zones within the one step of phase plate. The waves from two neighbounring zones have destructive interference in this point. The action of third zone offers uncompensate. The wave amplitude will be in three times and intensity in nine times less in point P_1 ($\mu_1 / \mu_3 = 9$, see Eq.(19)). In a similar manner we can be convinced that there are focal points in P_5 ,P_7... These points are at a distance f/5, f/7... from the plate, respectively.

The next approximation to kinoform of sperical lens is the phase plate witn three values of phase shift. If the transmittion of plate is given by:

$$\tau(\psi) = \begin{cases} 1, & 2\pi l < \psi \le \frac{2\pi}{3} + 2\pi l \\ \exp\left(i\frac{2\pi}{3}\right), & \frac{2\pi}{3} + 2\pi l < \psi \le \frac{4\pi}{3} + 2\pi l \\ \exp\left(i\frac{4\pi}{3}\right), & \frac{4\pi}{3} + 2\pi l < \psi \le 2\pi(l+1) \end{cases} \quad , \quad l = 0, \pm 1, \pm 2, \ldots \tag{23}$$

the diffraction efficiency in the first order μ_1 will be maximum. The dependence $h(\psi)$ for such plate is shown in Fig.2c. The diffraction efficiency of three-level phase plate follows from Eq.(5) and (7):

$$\mu_m = \frac{1 - cos\frac{2\pi m}{3}}{2\pi^2 m^2} \cdot \left\{3 + 4cos\frac{2\pi(m-1)}{3} + 2cos\frac{4\pi(m-1)}{3}\right\} \tag{24}$$

From Eq.(24) it is seen that in the diffraction order m=+1 a major part of the energy is concentrated:

$$\mu_{+1} = \frac{27}{4\pi^2} = 0.684 \tag{25}$$

It should be mentioned that the diffraction efficiencies μ_{+m} differ considerably from μ_{-m} (for example, $\mu_{+2} = 0$, $\mu_{-2} = 0.171$), and zeroth diffraction order is absent.

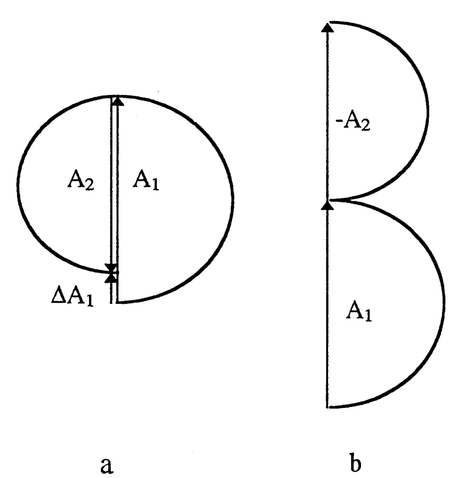

Fig.5. Vibration amplitudes from first and second Fresnel's zones without (a) and with Wood's plate.

As an example we consider the three-level plate with profile is shown in Fig.3b. It will be in fact the second after Wood's plate approximation to kinoform spherical lens kinoform. The action of this plate has been clearly explained by the vector diagram (Fig.6a). Either of the two Fresnel's zones is divided into three parts with equal square. The amplitudes of vibrations from these parts are shown by vectors OA, AB and BC.

The vectors AB and BC are rotated through $2\pi/3$ and $2*2\pi/3$, respectively (Fig.6a), because there is the stepwise profile with $h' = \lambda/3(n-1)$ (Fig.3b). As a result the amlitude of electric field at point P_1 increases $3\sqrt{3}/4$ times (Fig.5,6), and intensity 27/16 times (see Eq.(20), (25)) as compared with Wood's plate.

In a similar manner we consider the four-level phase plate. Its profile is shown in Fig.3c. This plate in fact is a third approximation to spherical lens kinoform. The vectors AB, BC and CD are rotated through $2\pi/4$, $2*2\pi/4$ and $3*\ 2\pi/4$, respectively (Fig.6b), because the plate has steps with height $h' = \lambda/4(n-1)$. Vibration from all steps in point P_1 will be in phase. The diffraction efficiency of such hologram will be $\mu_1 = 0.811$.

Thus, the N-level phase plate equalizes waves phases from all steps in the first focus (point P_1). The perimeter of inscribed N-sided poligon will tend to circle length with increasing of plate levels number. In the limiting case $N \to \infty$ the optical element with diffraction efficiency $\mu_1 = 1$ will be obtained. It is a kinoform of spherical lens (see dashed curve in Fig.3c).

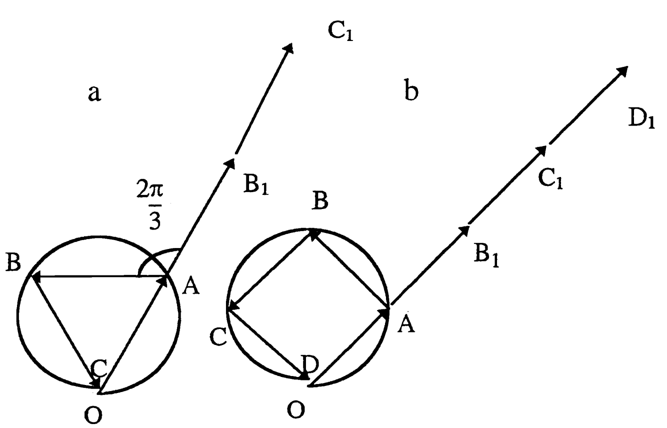

Fig.6. The changes of complex wave amplitudes from two Frenel's zones are introduced by three-levels (a) and four-levels (b) phase plates.

Similarly we can consider the phase plates with the rectilinear parallel zones. The widths of zones change with zone number as for discussed above elements (see Fig.3). Such phase plates will transform the incident plane wave to several converging and diverging cylindrical waves with amplitudes E_m given by Eq.(6). Converging waves show the light lines in focal planes. In Fig.4 these lines are perpendicular to figure plane and run along the points P_1, P_3, ... These plates are in fact the first approximations to cylindrical lens kinoform.

4. THE RELIEF FORMING METHODS.

Now we dwell on the methods forming the relief of multilevel phase elements[6,7].

Mechanical method. In this method the grooves cut immediately by diamond-tipped cutting tool. This method is employed to production axisymmetrical optical elements.

Photochemical methods. The substance of these methods are the variation of the chemical properties of material under exposure to light. For example, the regions of the surface with the different dissolving rates can be obtained. After chemical treatment the relief with the given profile will be produced. The gelatine with different additives (silver halide emulsions, bichromate gelatine) has such quality and is widely used for these purposes.

The light-sensitive polymer materials (such as polyvinil alcohol with addition of chromium salt, photoresist) are competitive with gelatine in the manufacture of relief with height about 5 - 7 μm.

Chalcogenite glassy semiconductors (such as arsenic selenide and arsenic sulphide) are used for production of optical elements for near IR and visible spectrum. (The relief height is about 1 μm).

And lastly, it is possible to produce the step-relief by photolithography methods.

In conclusion we note that it is possible to produce the transparent optical elements with the given phase profile using the change of refractive index. It can be reached, for examples, by implantation of foreign atoms and ions, photochemical reactions and photostimulation of phase transitions[7].

5. ACKNOWLEDGMENT

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