New teaching method to simplify Boolean logic functions using 3D Cubes

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ABSTRACT

In this work, a new teaching method to derive the Boolean logic functions is presented. Physical 3D cubes will be used to construct a 3D structure that will assist students in representing minterms of the Boolean function with 6 variables. The well-known grey code is applied to sort the cubes. This method will be used by educators in electrical, computer and communication engineering programs to explain Boolean logic function simplification using an interactive and easy way. The work also introduces different in-class activities that can be conducted to ensure better student engagement. The suggested 3D model is very simple and can be built using any transparent material like glass, acrylic or plastic, or solid colored material like wood or colored plastic. This work suggests different exercises to be implemented in-class to aid educators in introducing this kit to the students. This will result students in mastering the kit. Thus, students will have deeper knowledge and interest in digital logic design topics, which is an essential topic in any electrical, computer and communication engineering curriculums.

Keywords: Education, STEM, Educational Technology, Electrical Engineering Education, Engineering Students, Karnaugh Map, Boolean Function Simplification, Interactive environments.

1. INTRODUCTION

There have been several approaches suggested to simplify any given Boolean function to achieve gate level minimization. The most well-known method is Karnaugh maps (K-maps) introduced in 1953 by Maurice Karnaugh which was an improvement to the Veitch Chart that was presented by Edward W. Veitch's in 1952 [1], [2]. This method exploits human's pattern recognition capability to eliminate and group cells to represent the correct minimized logic expression. Solving any given Boolean expression requires the expression to be written first in its canonical form, which will result in having several minterms. These minterms are later on mapped on a Truth Table [3]. Afterwards, the minterms will be transferred to a 2-Dimentional grid, K-map. This grid utilizes grey code to order its cells, which are also referred to as minterms. Later on, those miterms will be grouped in an effective way to form the simplified Boolean expression.

The main reason to derive a simplified Boolean expression is to find the minimal number of Gates needed to implement any real-world logic requirement. Less number of gates means smaller size circuits. Having an expression with Sum-of-Product (SOP) will require AND gates that feeds into an OR Gate. On the other hand, having Product-of-Sum will require OR gates that feeds into an AND gate. K-maps can be used to reduce an expression with at most 4 variables. This method gets complex with an expression with more than 4 variables [3].

Another way to achieve gate level minimization was through applying Boolean Algebra method suggested by George Boole in 1854 [4]. The variables in the expression that utilizes Boolean Algebra are treated either as 0s or 1s (Binary Digits), or in other words, these variables are considered to be either True or False. Based on that concept, Boolean Algebra Laws and theorems are presented to assist in achieving a simplified expression. However, as the number of variables increase simplification process gets more challenging.

In 1952, another algorithm to minimize Boolean Functions was developed by Willard V. Quine and improved by McCluskey in 1956, called Quine–McCluskey algorithm (QMC) [5],[6]. This method is identical to K-maps method in functionality, but its tabular form makes it more appealing to be utilized in computer algorithms. However, this method has a high computational complexity.

From the simplifying methods listed above, K-maps is one of the prominent Digital Logic Design topics that has been given in many electrical, electronic information and computer curriculums. A teaching method was introduced in [7],

which modified the K-maps teaching method in such a way to give the instructors an advantage while introducing the topic during the lecture, and as well in the grading process while assessing students. Also, this method gave students an importunity to go back through their notes and ask questions on how the maps were constructed. In all, this method enhances the learning and grading process of using K-maps.

In addition, other several teaching tools were developed by researches to facilitate teaching K-maps, such that students will grasp the concept more easily and increase their interest in the topic. In [8], proposed a teaching module in nano-technology course that uses K-maps to introduce undergraduate students to nano-technology logic design concepts. The main aim from introducing this teaching methodology is to increase students' interest in the topic, thus enhancing their critical thinking, and further motivate them to investigate many emerging nano-technology devices.

In addition, many other forms of teaching approaches has been suggested to aid in introducing Elementary Digital Design course. In [9], authors have suggested a computer-aided software to aid students in solving any given Boolean expression using different minimizing techniques and build the correct combinational circuit. It provides a user-friendly environment due to its graphical interface. It is noted in [10] that adaptive and personalized learning technologies, open educational resources and learning analytics are the three most promising technologies that will impact engineering education.

In this study, a new teaching kit to be used by educators to teach Boolean functions representation will be introduced. The kit is a 3-D model referred as "The Cubes Model" to aid students in representing Boolean function minterms with 6 variables by sorting the cubes according to the grey code sequence. In addition, four different in-class exercises are proposed to aid educators in introducing this kit. The first exercise will lead students in finding the cube's position. The second exercise will help students in finding the minterm expression from the cubes' position, and as a result learn how the sides of the cube are labeled. The third one is an exercise that demonstrates how to sort all 64 cubes in the teaching kit. The last exercise will teach students how to find adjacent cubes and formulate the product term of the function Boolean logic expression.

2. THE CUBES MODEL

This model in Figure 1 is used to represent all minterms of any given 6 variable Boolean expression $G(A, B, C, D, E, F)$ in a 3D model. Where A is the most significant bit and F is the least significant bit. The model consists of three adjacent solid walls, where a total of 64 cubes will be placed on the model in an arrangement to construct the 3D model as shown in Figure 1. The solid walls can be made of wood (as suggested), plastic, acrylic or any other solid or transparent material.
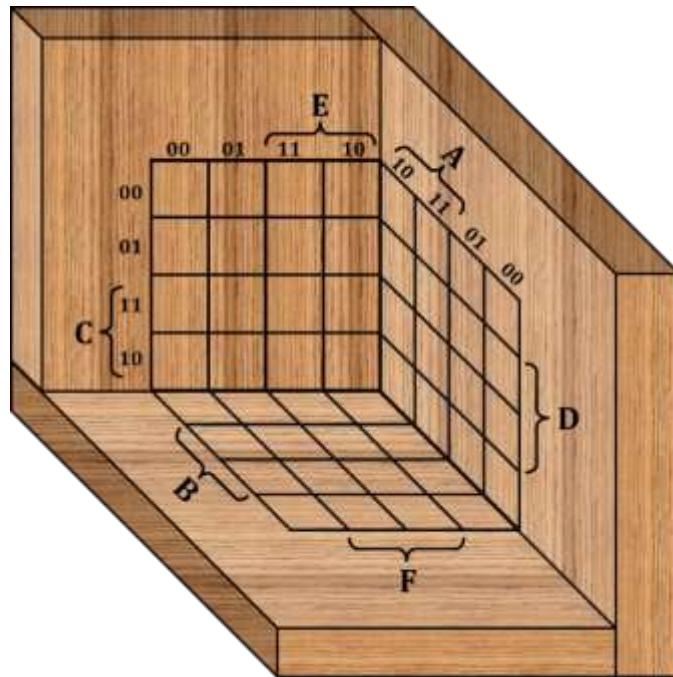

Figure 1 The structure of the 3D cubes model

The 3D cubes are numbered from 0 to 63 as shown in Figure 2 (a). Each cube has the position and the equivalent decimal minterm labeled on four out of the cube's six sides. The used as sides are shown in Figure 2 (b). The minterm decimal equivalent is labeled in the bottom side. The other three sides are labeled either with one of the combinations of AB, CD, and EF or their complements to represent the correct combination of a minterm. In other words, the side of the cube will be labeled with one of the four combinations $\overline{A}\,\overline{B}, \overline{A}B, AB$ or $A\overline{B}$, the front side of the cube will marked with one of the four combinations $\overline{C}\,\overline{D}, \overline{C}D, CD$ or $C\overline{D}$ and top side of the cube will denote one of the four combinations $\overline{E}\,\overline{F}, \overline{E}F, EF$ or $E\overline{F}$. Figure 2 (c) and (d) are showing minterm number 38 as an example. Two sides are left not labeled.
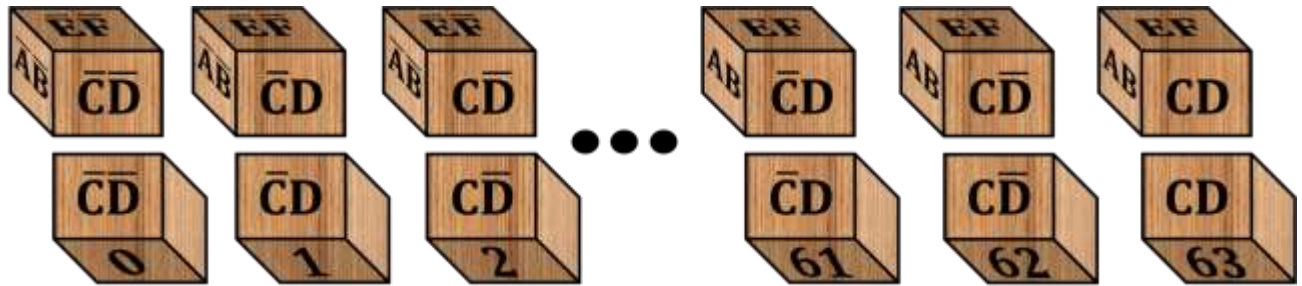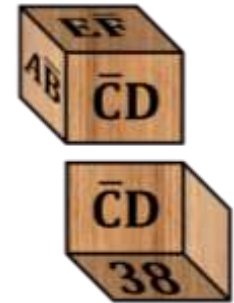

(a)
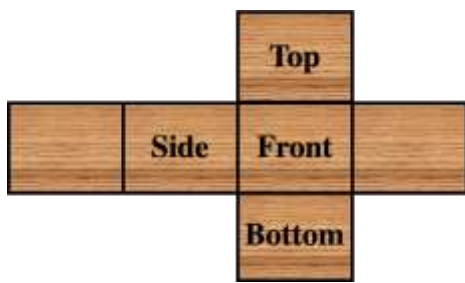
(b)
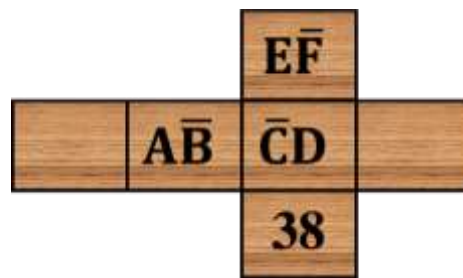
(c)

(d)

Figure 2 The labeling scheme of each Cube.

TABLE I presents all 64 minterms numbered from 0 to 63 showing each side of the cube with what letter combinations it will be labeled with.

TABLE I – CUBES' LABELS LIST

A	B	C	D	E	F	Side	Front	Top	Bottom	A	B	C	D	E	F	Side	Front	Top	Bottom
0	0	0	0	0	0	$\bar{A}\bar{B}$	$\bar{C}\bar{D}$	$\bar{E}\bar{F}$	0	1	0	0	0	0	0	$A\bar{B}$	$\bar{C}\bar{D}$	$\bar{E}\bar{F}$	32
0	0	0	0	0	1	$\bar{A}\bar{B}$	$\bar{C}\bar{D}$	$\bar{E}F$	1	1	0	0	0	0	1	$A\bar{B}$	$\bar{C}\bar{D}$	$\bar{E}F$	33
0	0	0	0	1	0	$\bar{A}\bar{B}$	$\bar{C}\bar{D}$	$E\bar{F}$	2	1	0	0	0	1	0	$A\bar{B}$	$\bar{C}\bar{D}$	$E\bar{F}$	34
0	0	0	0	1	1	$\bar{A}\bar{B}$	$\bar{C}\bar{D}$	EF	3	1	0	0	0	1	1	$A\bar{B}$	$\bar{C}\bar{D}$	EF	35
0	0	0	1	0	0	$\bar{A}\bar{B}$	$\bar{C}D$	$\bar{E}\bar{F}$	4	1	0	0	1	0	0	$A\bar{B}$	$\bar{C}D$	$\bar{E}\bar{F}$	36
0	0	0	1	0	1	$\bar{A}\bar{B}$	$\bar{C}D$	$\bar{E}F$	5	1	0	0	1	0	1	$A\bar{B}$	$\bar{C}D$	$\bar{E}F$	37
0	0	0	1	1	0	$\bar{A}\bar{B}$	$\bar{C}D$	$E\bar{F}$	6	1	0	0	1	1	0	$A\bar{B}$	$\bar{C}D$	$E\bar{F}$	38
0	0	0	1	1	1	$\bar{A}\bar{B}$	$\bar{C}D$	EF	7	1	0	0	1	1	1	$A\bar{B}$	$\bar{C}D$	EF	39
0	0	1	0	0	0	$\bar{A}\bar{B}$	$C\bar{D}$	$\bar{E}\bar{F}$	8	1	0	1	0	0	0	$A\bar{B}$	$C\bar{D}$	$\bar{E}\bar{F}$	40
0	0	1	0	0	1	$\bar{A}\bar{B}$	$C\bar{D}$	$\bar{E}F$	9	1	0	1	0	0	1	$A\bar{B}$	$C\bar{D}$	$\bar{E}F$	41
0	0	1	0	1	0	$\bar{A}\bar{B}$	$C\bar{D}$	$E\bar{F}$	10	1	0	1	0	1	0	$A\bar{B}$	$C\bar{D}$	$E\bar{F}$	42
0	0	1	0	1	1	$\bar{A}\bar{B}$	$C\bar{D}$	EF	11	1	0	1	0	1	1	$A\bar{B}$	$C\bar{D}$	EF	43
0	0	1	1	0	0	$\bar{A}\bar{B}$	CD	$\bar{E}\bar{F}$	12	1	0	1	1	0	0	$A\bar{B}$	CD	$\bar{E}\bar{F}$	44
0	0	1	1	0	1	$\bar{A}\bar{B}$	CD	$\bar{E}F$	13	1	0	1	1	0	1	$A\bar{B}$	CD	$\bar{E}F$	45
0	0	1	1	1	0	$\bar{A}\bar{B}$	CD	$E\bar{F}$	14	1	0	1	1	1	0	$A\bar{B}$	CD	$E\bar{F}$	46
0	0	1	1	1	1	$\bar{A}\bar{B}$	CD	EF	15	1	0	1	1	1	1	$A\bar{B}$	CD	EF	47
0	1	0	0	0	0	$\bar{A}B$	$\bar{C}\bar{D}$	$\bar{E}\bar{F}$	16	1	1	0	0	0	0	AB	$\bar{C}\bar{D}$	$\bar{E}\bar{F}$	48
0	1	0	0	0	1	$\bar{A}B$	$\bar{C}\bar{D}$	$\bar{E}F$	17	1	1	0	0	0	1	AB	$\bar{C}\bar{D}$	$\bar{E}F$	49
0	1	0	0	1	0	$\bar{A}B$	$\bar{C}\bar{D}$	$E\bar{F}$	18	1	1	0	0	1	0	AB	$\bar{C}\bar{D}$	$E\bar{F}$	50
0	1	0	0	1	1	$\bar{A}B$	$\bar{C}\bar{D}$	EF	19	1	1	0	0	1	1	AB	$\bar{C}\bar{D}$	EF	51
0	1	0	1	0	0	$\bar{A}B$	$\bar{C}D$	$\bar{E}\bar{F}$	20	1	1	0	1	0	0	AB	$\bar{C}D$	$\bar{E}\bar{F}$	52
0	1	0	1	0	1	$\bar{A}B$	$\bar{C}D$	$\bar{E}F$	21	1	1	0	1	0	1	AB	$\bar{C}D$	$\bar{E}F$	53
0	1	0	1	1	0	$\bar{A}B$	$\bar{C}D$	$E\bar{F}$	22	1	1	0	1	1	0	AB	$\bar{C}D$	$E\bar{F}$	54
0	1	0	1	1	1	$\bar{A}B$	$\bar{C}D$	EF	23	1	1	0	1	1	1	AB	$\bar{C}D$	EF	55
0	1	1	0	0	0	$\bar{A}B$	$C\bar{D}$	$\bar{E}\bar{F}$	24	1	1	1	0	0	0	AB	$C\bar{D}$	$\bar{E}\bar{F}$	56
0	1	1	0	0	1	$\bar{A}B$	$C\bar{D}$	$\bar{E}F$	25	1	1	1	0	0	1	AB	$C\bar{D}$	$\bar{E}F$	57
0	1	1	0	1	0	$\bar{A}B$	$C\bar{D}$	$E\bar{F}$	26	1	1	1	0	1	0	AB	$C\bar{D}$	$E\bar{F}$	58
0	1	1	0	1	1	$\bar{A}B$	$C\bar{D}$	EF	27	1	1	1	0	1	1	AB	$C\bar{D}$	EF	59
0	1	1	1	0	0	$\bar{A}B$	CD	$\bar{E}\bar{F}$	28	1	1	1	1	0	0	AB	CD	$\bar{E}\bar{F}$	60
0	1	1	1	0	1	$\bar{A}B$	CD	$\bar{E}F$	29	1	1	1	1	0	1	AB	CD	$\bar{E}F$	61
0	1	1	1	1	0	$\bar{A}B$	CD	$E\bar{F}$	30	1	1	1	1	1	0	AB	CD	$E\bar{F}$	62
0	1	1	1	1	1	$\bar{A}B$	CD	EF	31	1	1	1	1	1	1	AB	CD	EF	63

The 64 cubes will be arranged utilizing the grey code sequence, such that adjacent cubes will differ by only one bit. Figure 3 (a) and (b) represents how cubes will be arranged.
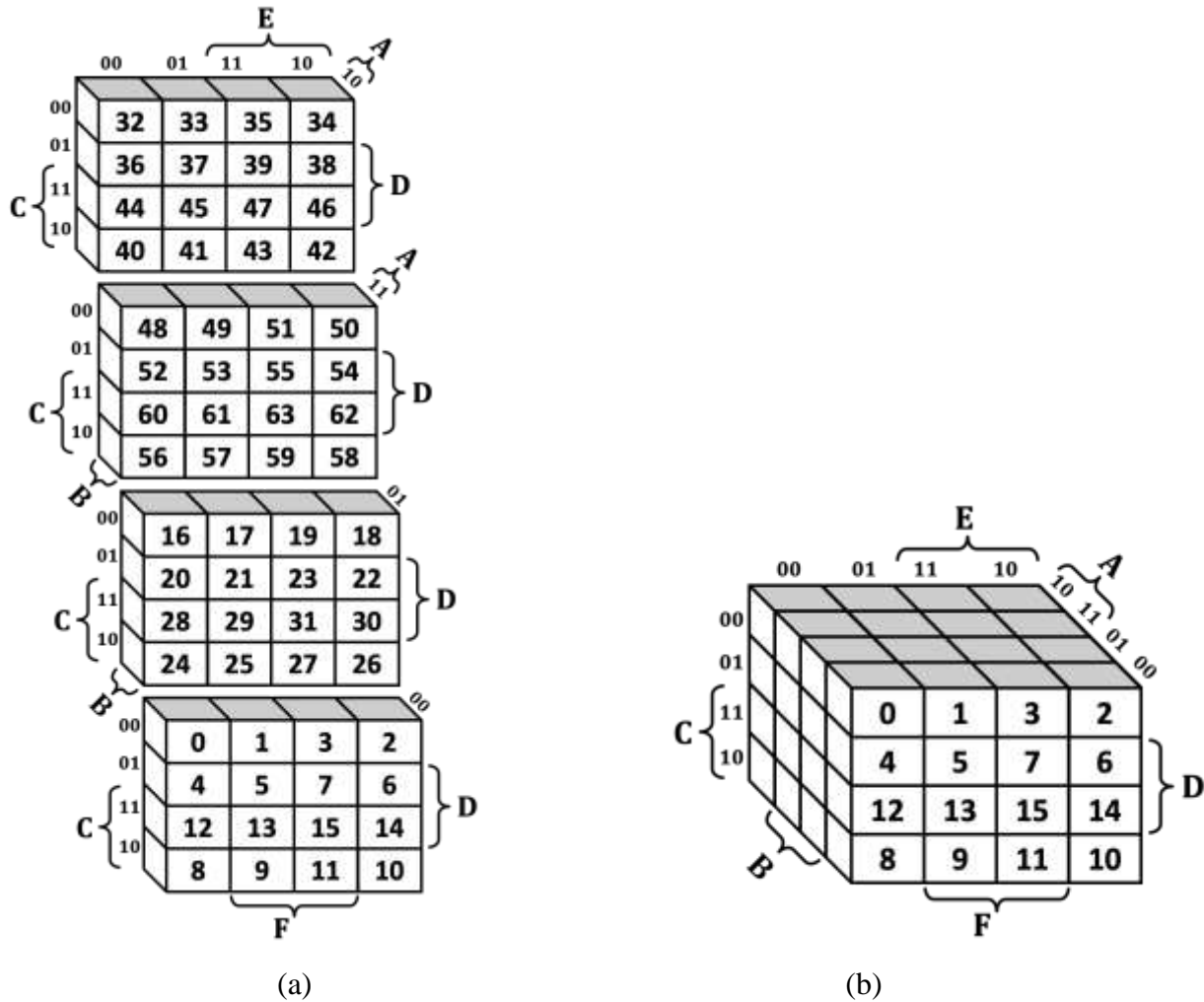
Figure 3 Cubes arrangement

3. CLASSROOM EXERCISES

This model is proposed to students as a teaching tool to aid them in representing Boolean Logic Functions with 6 variables. To achieve this objective, several exercises should be presented to the students during the lecture. Those exercises are listed below.

3.1 Finding the Cube's Position

In this exercise students will practice finding the bottom side number of the cube. First, cubes are handed to students, covering the bottom side. Students will find the minterm value that the cube represents by checking the value of the sides of the cube in the following order: Side-Front-Top. Afterwards, students will convert the obtained minterm to its binary number format representation. Where the bar ($\overline{X}$) means that the binary format is 0. Otherwise, it is 1, where X can be any letter from A to F. Finally, the binary number will be converted to its decimal format. This decimal number will represent the bottom number value. Thus, will lead students to find its exact position.

For example, given the cube in Figure 2 (c). Consider that the bottom side is unknown for the students. By checking the sides in the previous mentioned order, student will find that the cube has the following minterm: $A\ \overline{B}\ \overline{C}\ D\ E\ \overline{F}$. The binary equivalent of the minterm is: 100110 which is equal to 38 in decimal. The number 38 will be the value of the bottom side which represents the position of the cube.

Each student can be given more than one cube during the session to help them practice more in finding the cube's position. At the beginning it can be done as group work and then it can be practiced individually.

3.2 Labeling Cubes' Sides

With this exercise, students will learn how to label the side, front and top of the cube by only giving them the number in the bottom of the cube which denotes the cube's position. The given number is in in decimal format. First students will convert the number given to binary and then will represent the obtained value to its actual minterm. Whenever the number is a binary 0 it means that the letter value will be expressed with a bar ($\overline{X}$), where X can be any letter from A to F. The minterm founded will label the sides of the cube in the previously mentioned order.

For example, given the cube in Figure 2 (b). Consider that the side, front and top are unknown for the students. Students are provided with the bottom value which is 38. Students will convert the decimal value 38 into binary by doing the following process in Figure 4. The obtained converted value from Figure 2 (b) is 100110. This means the minterm represented by this cube is: $A\overline{B}\,\overline{C}DE\overline{F}$.

$$
\begin{array}{c|c}
38 & 0 \\
19 & 1 \\
9 & 1 \\
4 & 0 \\
2 & 0 \\
1 & 1
\end{array}
\quad \rightarrow \quad (\, 100110 \,)_2
$$

Figure 4 Decimal to binary conversion

Like in the previous exercise, each student can be provided with more than one cube during the session to help them practice more in labeling the sides of the cube in individual and group work settings.

3.3 Cubes Sorting

After students have gained enough experience in labeling and finding the position of the cube, students will practice on sorting the cubes in the correct arrangement, following the grey code sequence. Figure 3 (a) represents the sequence of the correct cubes' positioning.

Each student will be handed with "The Cubes Model" teaching kit, having all 64 cubes disarranged. Students will work on arranging the cubes in the model according to the correct sequence. This exercise can be done in individual and group work settings.

3.4 Checking Cubes Adjacency

Since the cubes arrangement follows the grey code sequence, it is very essential for students to practice how to find which cubes are adjacent to one another. Grey code sequence means that the difference between adjacent values is only 1 bit. In this exercise, students will be given first two cubes. Then, students will check the minterms of the cubes by looking at the side, front and top of the cube. If the difference in minterms is only in one bit this means the cubes are adjacent. Otherwise, they are not adjacent.

For example, consider cubes with numbers 23 and 55 are handed to the students. The minterm of the first cube is: $\overline{A}B\overline{C}DEF$ and the second's cube minterm is $AB\overline{C}DEF$. Comparing both miterms will give us that the there is a difference in the most significant bit. This means that the difference is only in one bit. Thus, the two cubes are adjacent. If needed to formulate the simplified expression of those two adjacent cubes, it is: $B\overline{C}DEF$. Another example, let's consider that cubes with bottom numbers 15 and 16 are handed to students. The minterm of the first cube is: $\overline{A}\,\overline{B}CDEF$ and the second's cube minterm is $\overline{A}B\overline{C}\,\overline{D}\,\overline{E}\,\overline{F}$. Comparing both miterms will give us that the only the most significant bit is similar while the rest of the bits are totally the opposite. This means the difference is in more than one bit. Thus, those cubes are not adjacent.

Another exercise that can be done is by giving students cubes to check their adjacency. To know if the four cubes are adjacent, there should be four common variables in all the handed cubes. For example, TABLE *II* shows four cubes with the following positions: 0, 16, 48 and 32. Their minterms and binary format is presented for each in the table. By comparing columns, A, B, C, D, E and F, which are the binary format, it is clear that the four cubes share four common columns which are C, D, E and F. All of the bits are binary 0. Therefore, those four cubes are adjacent and can be reduced and simplified to $\overline{C}\,\overline{D}\,\overline{E}\,\overline{F}$.

TABLE II – FORMING A PRODUCT TERM OF 4 VARIABLES

A	B	C	D	E	F	Side	Front	Top	Bottom
0	0	0	0	0	0	$\overline{A}\,\overline{B}$	$\overline{C}\overline{D}$	$\overline{E}\overline{F}$	**0**
0	1	0	0	0	0	$\overline{A}$ B	$\overline{C}\overline{D}$	$\overline{E}\overline{F}$	**16**
1	1	0	0	0	0	A B	$\overline{C}\overline{D}$	$\overline{E}\overline{F}$	**48**
1	0	0	0	0	0	A $\overline{B}$	$\overline{C}\overline{D}$	$\overline{E}\overline{F}$	**32**

A third exercise is to hand students eight cubes to check their adjacency. To know if the eight cubes are adjacent, they should share three common bits. As an example, TABLE III presents eight cubes with their minterms and binary numbers expressed. Columns A to F represents the binary format for of those cubes. Comparing those columns, it is shown that columns B, C and D are filled with binary number 0. Since these columns are similar means that they are adjacent. Also, if asked to express the simplified form of those adjacent cubes, the correct answer will be $\overline{B}\,\overline{C}\,\overline{D}$.

TABLE III – FORMING A PRODUCT TERM OF 3 VARIABLES

A	B	C	D	E	F	Side	Front	Top	Bottom
0	0	0	0	0	0	$\overline{A}\,\overline{B}$	$\overline{C}\overline{D}$	$\overline{E}\overline{F}$	**0**
0	0	0	0	0	1	$\overline{A}\,\overline{B}$	$\overline{C}\overline{D}$	$\overline{E}$F	**1**
0	0	0	0	1	1	$\overline{A}\,\overline{B}$	$\overline{C}\overline{D}$	EF	**3**
0	0	0	0	1	0	$\overline{A}\,\overline{B}$	$\overline{C}\overline{D}$	E$\overline{F}$	**2**
1	0	0	0	0	0	A $\overline{B}$	$\overline{C}\overline{D}$	$\overline{E}\overline{F}$	**32**
1	0	0	0	0	1	A $\overline{B}$	$\overline{C}\overline{D}$	$\overline{E}$F	**33**
1	0	0	0	1	1	A $\overline{B}$	$\overline{C}\overline{D}$	EF	**35**
1	0	0	0	1	0	A $\overline{B}$	$\overline{C}\overline{D}$	E$\overline{F}$	**34**

Last but not least, students also can recognize how to know if 16 cubes are adjacent or not. This is done by checking if those cubes share two common bits. Table presents the minterms and binary formats of 16 cubes. By checking the bits present in Columns A to F, it is found that columns D and F are always 1. This means that those cubes are adjacent and can be simplified to DF.

In summary, each student will be handed with two cubes as a start, to find their minterms and compare. Later on, with experience, students might be given more than two cubes to find which ones are adjacent by comparing their minterms. Also, students can start learning how to formulate the product term of the adjacent cubes. Like in the mentioned previous exercises, these exercises can be done in individual and in group work settings.

TABLE IV – FORMING A PRODUCT TERM OF 2 VARIABLES

A	B	C	D	E	F	Side	Front	Top	Bottom
0	0	0	1	0	1	$\bar{A}\bar{B}$	$\bar{C}D$	$\bar{E}F$	**5**
0	0	0	1	1	1	$\bar{A}\bar{B}$	$\bar{C}D$	EF	**7**
0	0	1	1	1	1	$\bar{A}\bar{B}$	CD	EF	**15**
0	0	1	1	0	1	$\bar{A}\bar{B}$	CD	$\bar{E}F$	**13**
0	1	0	1	0	1	$\bar{A}B$	$\bar{C}D$	$\bar{E}F$	**21**
0	1	0	1	1	1	$\bar{A}B$	$\bar{C}D$	EF	**23**
0	1	1	1	1	1	$\bar{A}B$	CD	EF	**31**
0	1	1	1	0	1	$\bar{A}B$	CD	$\bar{E}F$	**29**
1	0	0	1	0	1	$A\bar{B}$	$\bar{C}D$	$\bar{E}F$	**37**
1	0	0	1	1	1	$A\bar{B}$	$\bar{C}D$	EF	**39**
1	0	1	1	1	1	$A\bar{B}$	CD	EF	**47**
1	0	1	1	0	1	$A\bar{B}$	CD	$\bar{E}F$	**45**
1	1	0	1	0	1	AB	$\bar{C}D$	$\bar{E}F$	**53**
1	1	0	1	1	1	AB	$\bar{C}D$	EF	**55**
1	1	1	1	1	1	AB	CD	EF	**63**
1	1	1	1	0	1	AB	CD	$\bar{E}F$	**61**

4. CONCLUSION

This work presented the teaching kit "The Cubes Model" as a 3D tool to aid students in representing minterms of Boolean logic functions. The structure of the model consists of three solid walls, were 64 cubes arranged in a way to represent a 6 variable Boolean function. Students will use different senses and their recognition capabilities to sort and arrange the cubes. Four different exercises were proposed in this study. The first two exercises will lead students in knowing how to find the cubes' positions and minterms expressions, and thus help them to better understand how cubes' sides are labeled. The third exercise will teach students how to sort and fix up the cubes in the correct way on the model according to grey code arrangement. The fourth exercise will give students a better understanding in realizing adjacent cubes, thus learning how to express the simplified expression of those adjacent cubes. All exercise can be done in individual and group work settings.

Since Boolean logic function is a common topic found in several engineering curriculums, this kit has an important impact in facilitating education in several engineering fields, where both educators and students will benefit from using it. Educators will find it as an easy teaching tool to be introduced to students. Thus, resulting in an increase student's interactivity, which is the main objective of every educator. On the other hand, students will use more than one sense which will positively impact their learning process. Thus, this will lead students to better understand the topic. In conclusion, programs that will benefit from using this kit, but not limited to, are electrical, computer and communication engineering programs.

For future work, a computer-based software version of this kit is suggested to aid educators in the teaching process and help students better understand Boolean logic functions. Moreover, additional features maybe added to the software such as auto grading tools to help students in practicing and do self-assessments. Besides, additional interactive features can be added to the suggested 3D model, such as different sounds or lightning effects, to notify students if any incorrect positioning of any cube is present. Thus, leading them to learn independently.

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