

Optical characterization of volume-scattering backgrounds

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ABSTRACT

Scattering media act in many situations as backgrounds in target recognition and remote sensing and an accurate method for their characterization is highly desirable. The use of light sources with short temporal coherence produces the depth resolution needed for this purpose. The low coherence interferometry has been used, for long time, as a filter to suppress multiple light scattering and preserve the single-scattering characterized by well defined scattering angles and polarization properties. Recently, the low-coherence interferometry was successfully applied to multiple light scattering regime. The signal obtained from such a measurement relates directly to the optical path-length distribution of the backscattered light and, therefore, comprehensively characterizes the scattering system. The path-length resolved backscattering defines the scattering properties of the medium and its shape has distinct features for single and multiple-scattering regimes. In our experiments, the path-length domain is sampled with a resolution equivalent to 30 fs in conventional time-of-flight measurements. We will show that the transition domain between single and multiple scattering can be fully characterized using this methodology and that single scattering information can be successfully retrieved even in the presence of a strong multiple scattering component.

1 Introduction

In many practical situations, the problem of target recognition becomes a difficult task due to the presence of various screening media surrounding the target, which act as backgrounds. Optical methods have been proven to have a great potential for remote sensing applications and different techniques have been proposed for both static and moving targets¹⁻³. An important configuration for optical investigations is the monostatic one, where the source and the detector are spatially overlapped. This geometry offers certain practical advantages and will receive our special attention throughout the present paper. Isolating the signal from the background noise remains a problem of major concern in such applications. Often the backgrounds consist of random distributions particles, droplets, etc., which attenuate the propagating light towards and from the target through processes of absorption and scattering, as illustrated in Fig. 1.

Therefore isolating the field that returns unscattered from the target becomes more difficult as the density of the background increases. This is because the light suffers many scattering events upon propagation, which dramatically lowers the strength of the component that propagates forwardly. This unscattered (ballistic) fraction of the propagating waves is of utmost importance for target recognition

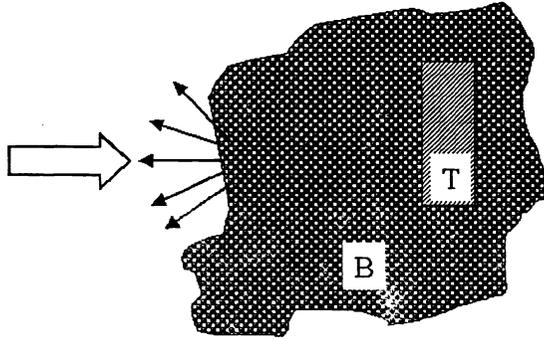


Figure 1: Target (T) surrounded by a scattering background (B) under optical investigation

and remote sensing, since it is characterized by well defined direction of propagation and carries the information about the target.

In this paper, we investigate the potential of using low-coherence light for background characterization. The experiments are performed at a small scale in the laboratory, but the principle of the technique can be applied at a larger scale, such as in open field situations. In Section 2 we show that the coherence properties of the light allows isolating very weak ballistic signals propagating through dense distributions of particles. This type of measurements offers information about the medium, through its mean free path. In addition, our experimental results show that, due to its broad optical spectrum, the light attenuates in a non-trivial manner through the scattering medium, which can no longer be described by a simple Lambert-Beer equation. In Section 3, the low-coherence is used to measure the pathlength distribution of backscattered waves from a dense distribution of particles. This technique, referred to as optical pathlength spectroscopy,⁴⁵ makes use of the diffuse backscattered light to infer information about the scattering medium in terms of its transport mean free path. Both configurations are characterized by high dynamic range and temporal resolution and prove that the interference with low-coherence light can be used to characterize backgrounds in the context of remote sensing applications.

2 Background characterization using ballistic propagation of low-coherence light

The customary description of the attenuation suffered by a monochromatic light beam propagating in a scattering, non-absorbing medium is given by the Lambert-Beer law: $P = P_0 \exp(-L/l_s)$, where P_0 is the initial power, P is the remaining power after traveling a distance L , while l_s is the scattering mean free path that characterizes the medium. When absorption is present, the same exponential decay applies, but the decay rate $1/l_s$ is replaced by $1/l_s + 1/l_a$, with l_a the absorption length. Deviations from the Lambert-Beer law due to multiple scattering have been quantified⁶ and corrections have been proposed.⁷ Modifications of classical Lambert-Beer dependence of light attenuation have also been observed in the case of highly-correlated scattering systems.⁸ We propose a novel method for measuring the ballistic component of light that travels through dense scattering media. The accuracy of this technique allows, for the first time, to our knowledge, to investigate fine details of the attenuation process, such as the influence of the broad optical spectrum on the ballistic light attenuation and the relationship of this dependence with the scatterer size. Such attenuation measurements offer the

possibility to obtain the mean free path l_s of the scattering medium. This information should be useful in applications that involve target recognition in the presence of scattering backgrounds. The method involves a low-coherence light source in an interferometric geometry. The experimental setup, is depicted in Figure 2. The broadband source is an LED that emits light with a central wavelength of

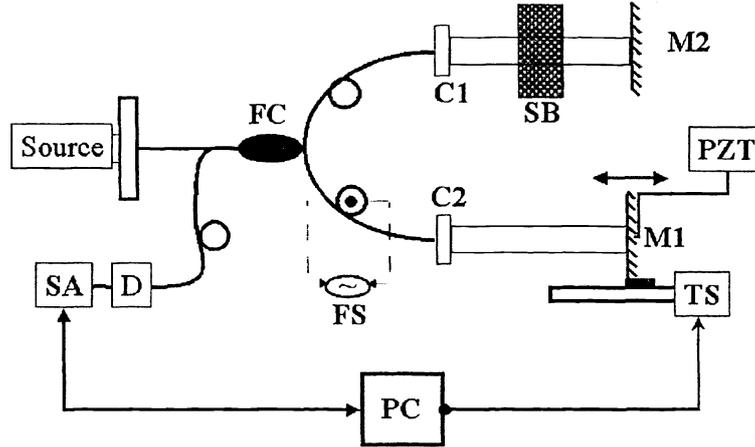


Figure 2: Experimental setup for ballistic waves attenuation through a scattering background. FC fiber coupler; C1, C2 collimators; SB scattering background; M1, M2 mirrors; PZT piezoelectric transducer; TS translation stage; FS fiber stretcher; D detector; SA spectrum analyzer; PC personal computer.

$1.33\mu m$ and a FWHM bandwidth of $60nm$. The light emitted by the LED is coupled into a single-mode optical fiber that enters the 2×2 fiber coupler FC. The beam is split here into reference and probe arm. On the reference arm, the light at the end of the fiber is collimated and sent to the scanning mirror $M1$ and, on the probe arm, the beam is sent to a second mirror $M2$. The system under investigation is placed between the coupler and mirror $M2$. The fraction of light passes through the medium and is attenuated accordingly due to the scattering process. However, the unscattered component will be reflected by the mirror $M2$ and forced to propagate through the medium a second time. The ballistic (unscattered) fraction is now transmitted by the coupler back into the fiber and directed to the detector where is combined with the light reflected by the reference mirror $M1$. When the optical pathlength of the reference and probe arms match within the coherence length of the source, the two fields interfere. The optical length of the reference arm can be adjusted by moving the mirror $M1$ placed on the computerized translation stage TS . Whenever the light on the probe arm travels the same optical distance as in the reference arm, a maximum of interference occur at the detector. Due to the low-coherence of the source, the interference will manifest on a narrow spatial interval around the maximum. In our case, the coherence length of the source is about $10\mu m$, which allows a precise isolation of the signal propagating unscattered through the medium. The expression for the irradiance at the detector, as a function of the optical pathlength set by the reference mirror, takes the form:

$$I_d = I_b + I_{ref} + 2 \cdot \sqrt{I_b} \cdot \sqrt{I_{ref}} \cdot \cos(2\pi \cdot \Delta s / \lambda), \quad (1)$$

where I_d , I_b , and I_{ref} are the detected, ballistic, and reference irradiance, respectively. The optical path difference between the scattered and reference field is denoted by Δs and λ is the central wavelength. Measuring the modulus of the interference peak, one is able to determine the ballistic component of light and, therefore, extract information about the attenuation properties of the scattering medium. In our experiments the light is phase-modulated either by the fiber stretcher attached to the reference fiber or by the piezo-electric transducer, which vibrates the mirror $M1$. Therefore, the interference signal is read by the spectrum analyzer (SA) at the modulation frequency, which in our measurements

had a value of 13 kHz. A personal computer (PC) is used to both translate the stage on the reference arm and to record the data from the spectrum analyzer.

In the following, we investigate the potential of this geometry for ballistic light measuring in the presence of strong multiple scattering. When a scattering medium is placed between $C2$ and $M2$, the magnitude of the interference peak describes quantitatively the beam attenuation due to the scattering process. Using the heterodyne detection, we are able to evaluate this attenuation over a dynamic range of more than $80dB$ and the short coherence length of the light offers the temporal selectivity necessary to isolate the early-arriving ballistic waves. As it is well known, angular filtering is one of the main concerns in measurements of ballistic light⁹⁻¹¹. The measured FOV for the present configuration was 0.85 mrad, which proves the high selectivity of the unscattered light. Another characteristic of this configuration is that it allows testing the medium with a collimated beam over a large area, and, therefore, a better average across the sample is obtained. As all fiber optic-based systems, it has the advantage of stability and versatility over other open-beam geometries. We should also mention that the alignment and position of the sample under test are not critical for this geometry.

To prove the potential of this technique for ballistic light measurement, we performed experiments on suspensions of polystyrene microspheres of different particle sizes and concentrations. For each particle size, various concentrations were obtained by mixing standard suspensions with deionized water, and the measurements were taken through cuvettes of $1cm$ thickness. Fig. 3 (a, b, c) shows the transmission results for scattering media of three particle sizes: $0.121\mu m$, $0.356\mu m$, and $3.135\mu m$, for which the scattering cross-sections, σ_s , are $9.78 \cdot 10^{-14}cm^2$, $5.0 \cdot 10^{-11}cm^2$, and $2.83 \cdot 10^{-7}cm^2$, respectively. For each concentration, the scattering mean free path was calculated using the formula $l_s = 1/(N\sigma_s)$, where N is the particle concentration. Each data set was normalized with respect to the value of transmission through deionized water placed in the same cuvette to account for the effect of absorption in water, which has a constant value for all the samples. As it can be seen from Fig 3, the experimental decay sharply ends at the noise level of the device, which is an indication that the filtering of scattered light is virtually complete due to both the coherence and angular gating. However, a disagreement between the experimental data and the expectation of the Lambert-Beer law is observed.

The deviation from the negative exponential behavior becomes more important for samples of smaller particle size, as can be observed in Fig. 3 (a, b). This particle size dependence of the ballistic light attenuation is unexpected and a more refined interpretation is needed to explain such a behavior. The Lambert-Beer law in its form presented earlier applies to situations where the light source is monochromatic. In our measurements, however, the spectral density of the source approaches a Gaussian with a FWHM of about $60nm$, centered at $\lambda_0 = 1.33\mu m$. In this case, the transmission coefficient, defined as the ratio between the transmitted and the incident power, takes the form:

$$T_{\Delta\lambda}(\lambda_0) = \int_{-\infty}^{+\infty} S_{\Delta\lambda}(\lambda - \lambda_0) \cdot \exp[-N\sigma_s(\lambda) \cdot L] d\lambda \quad (2)$$

The equation 2 asserts that the overall transmitted light is given by the summation of the contributions from all the frequency components, weighted by the initial optical spectrum and it takes the form of a convolution between the Lambert-Beer transmission and the incident spectral density $S_{\Delta\lambda}(\lambda - \lambda_0)$, of central frequency λ_0 and width $\Delta\lambda$. It can be easily seen in Fig. 3 that Eq. 2 describes much better the experimental results for all the particle sizes. The correction brought to the Lambert-Beer law by Eq. 2 has an increasing importance as the particle size decreases and becomes negligible for large particles. The physical interpretation for this result is that the scattering cross-section depends stronger on the size parameter $2\pi R/\lambda$, for smaller values of the particle radius R . Therefore, in this range of size parameters, the particles are characterized by different scattering cross sections corresponding to

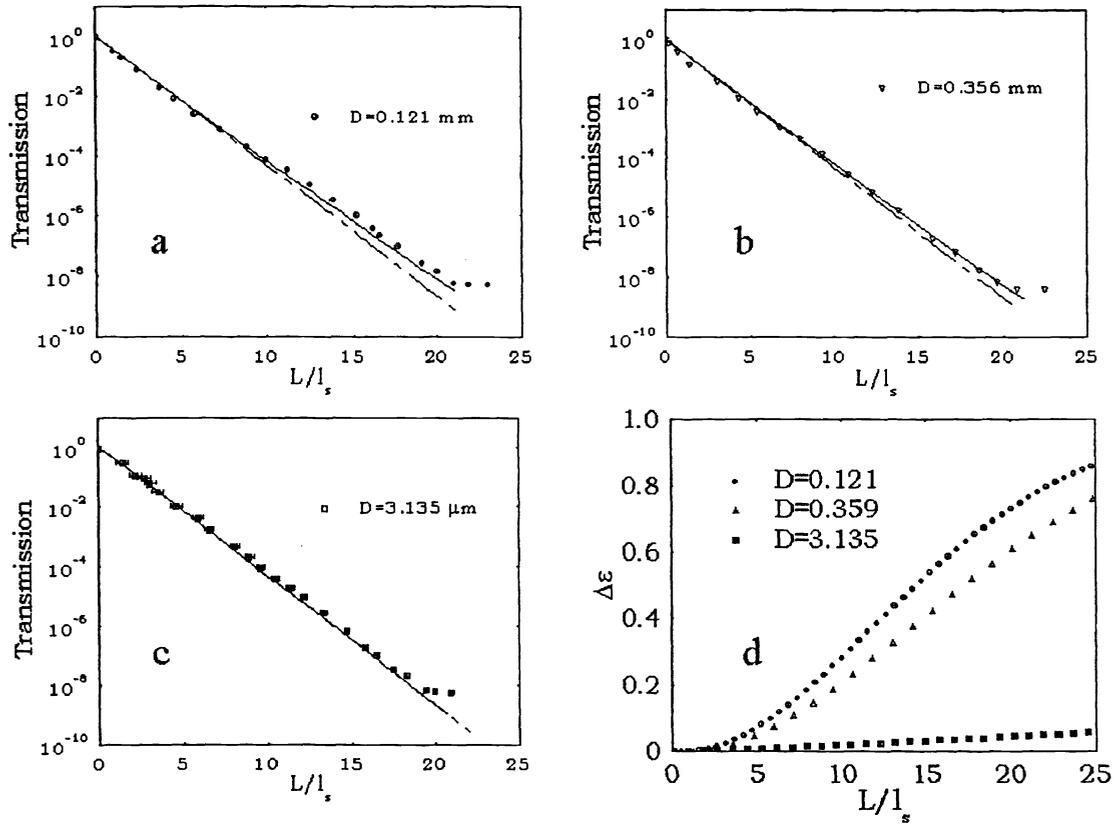


Figure 3: Experimental results for ballistic attenuation through media of different particle sizes, as indicated. In a, b, c, the dashed lines represent exponential decays; the continuous lines are given by Eq. 2. In d, $\Delta\epsilon$ is the relative difference between the Lambert-Beer law and Eq. 2.

each frequency component of the optical spectrum. In the domain of large particle sizes, the cross sections associated with individual spectral components do not change significantly over the incident spectrum and, thus, the overall attenuation approaches the negative exponential decay, as in the case of monochromatic light.

Compared with the confocal geometry, our technique has the advantage of the coherence gate, which allows isolating the early arriving ballistic light and, in addition, the angular filtering does not require additional lens-pinhole systems. It should be pointed out here that the time-gated measurements can also provide the pathlength selectivity necessary to isolate the ballistic light component. A major drawback of the ultrafast gating is that it requires extensive experimental infrastructure systems. The technique allows isolating the ballistic light component that propagates through strongly scattering media. Evaluating the attenuation process over a high dynamic range, the scattering and absorption properties of the medium can be quantified. The highest density of media that can be investigated by this method is limited by the dynamic range of the measurement, since the useful signal is provided by the portion of light that passes through the medium unscattered. In the following, we show that by using the diffusive light as the information carrier, scattering systems of higher concentration can be investigated.

3 Characterization of diffusive backgrounds using optical pathlength spectroscopy

Low-coherence interferometry (LCI), as described earlier, has become a widely used technique for various applications. So far, LCI has been used as a filter that suppresses the multiple light scattering and preserves the single scattering component characterized by well defined scattering angles and polarization.¹² We show, that LCI technique can also be used to investigate the multiple scattering regime of waves propagation and we discuss some characteristics which make this approach particularly appealing for applications, such as characterization of dense backgrounds.

A multiple scattering regime is usually associated with waves propagation through optically dense random systems and is commonly described in terms of a diffusion equation. This is an approximation for energy transport where isotropic elastic scattering and wave propagation at a constant group velocity are considered, while the polarization and interference effects are neglected.¹³ The diffusive wave propagation depends on the specific scattering geometry and is characterized by the probability density $P(s)$ of optical pathlengths through the medium. In general, $P(s)$ can be theoretically estimated for different experimental configurations but, so far, direct experimental studies regarding optical pathlengths distribution have been limited to investigations of temporal broadening of short light pulses that propagate diffusively.¹⁴ The time t necessary for the optical wave to propagate along a path of length s is simply given by $t = s/v$, where v is the average velocity of energy transport. The steady-state transport mean free path (l_t) relates to the dynamic diffusion coefficient D by considering a constant energy transport velocity $v = 3D/l_t$. In steady-state conditions, l_t depends on both number density of scatterers and size and shape of each individual scatterer: $l_t = [N\sigma_s(1-g)]^{-1}$, where N is the number density of scatterers, σ_s is the cross section of a single scattering event and g is the average cosine of the scattering angle. For media of finite thickness, the condition under which the diffusion theory is generally valid is $l_t/L \ll 1$, where L is the thickness of the random medium. The diffusion approximation has been questioned in the past and experiments of time-resolved diffuse transmission showed that it becomes less and less reliable when the thickness of the sample decreases and the anisotropy factor increases.¹⁵ However, it has been shown that when internal reflections at the boundary and scattering anisotropy are properly taken into account, the diffusion predictions are accurate for samples as thin as about $5l_t$.¹⁶ Recently, Lemieux et al. expanded the applicability limits of the diffusion approach by using a telegrapher equation which takes into account the ballistic transport and modeling the scattering anisotropy in terms of a field concentration discontinuity at the source point.¹⁷

We propose a novel approach to investigate the wave propagation through random media. Based on LCI principle, the new technique, called optical pathlength spectroscopy (OPS), infers directly the pathlengths distribution $P(s)$ of waves scattered by a random medium. We present OPS experiments in a backscattering geometry which is appealing for a variety of applications, but this is by no means a restriction for the principle of the proposed method. It is worth noting that the information provided by OPS is somehow similar to that obtained in time-resolved reflectance measurements, where the diffusion approximation makes a reasonable description of the experimental data.

In the present LCI geometry, depicted in Fig. 4, light from a broad-bandwidth source is first split into probe and reference beams which are both retroreflected from a targeted scattering medium and from a reference mirror, respectively, and are subsequently recombined to generate an interference signal. Assuming quasi-monochromatic optical fields ($\Delta\lambda/\lambda \ll 1$), the detected intensity has the simple form of Eq. 1. Two conditions are needed in order to obtain interference maxima: i) Δs to be a multiple of wavelength and ii) $|\Delta s| < l_{coh}$, where l_{coh} is the coherence length of the source. In the present OPS configuration, I_b corresponds now to the reflectance of a multiple scattering medium. In LCI, only the class of waves that have traveled an optical distance which corresponds to the length

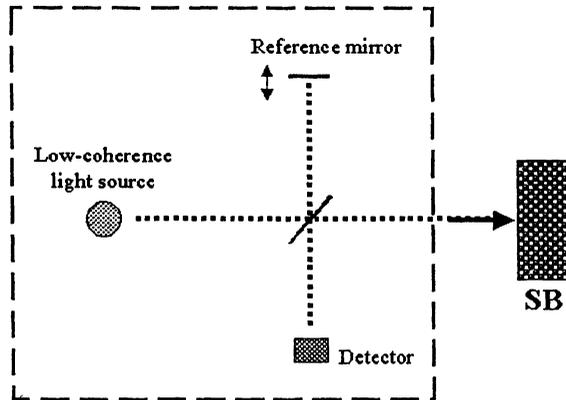


Figure 4: Backscattering configuration for optical pathlength spectroscopy.

of the reference arm is able to produce fringes and is, therefore, detected. In the optical pathlengths domain, the interferometer acts as a bandpass filter with a bandwidth given by the coherence length of the source. Accordingly, the shorter the coherence length, the narrower the optical pathlengths interval of backscattered light that will produce detectable fringes. Now, if we let the reference mirror sweep the reference arm, waves with different optical pathlengths through the medium are detected and an optical pathlengths distribution is reconstructed. As the reference mirror is moved, the peak of the beat signal produced is proportional to the squared root of the optical reflectance, the quantity of interest in our backscattering geometry. We note that the OPS approach is experimentally limited by the fact that the signal corresponding to long paths within the medium is weak and a large dynamic range is needed for accurate measurements in the tails of pathlength distributions. However, as opposed to dynamic techniques, there is no need for sophisticated time-of-flight configurations and the measurement can be taken over longer periods of time to obtain a better average. The reflectance signal is typically averaged over 100 successive scans which significantly improves the signal to noise ratio.

We present OPS data obtained on suspensions of polystyrene microspheres in water, with various volume concentrations. The low-coherence interferometer used for our studies has a light emitting diode as a source with a coherence length of $10\mu m$ and a broad wavelength band centered at $1330nm$. The same single mode optical fiber is used to send and collect the light from the random medium and throughout our experiments, the dimensions of the samples are much larger than the transport mean free path and the illumination geometry is such that the scattering system can be treated as a semi-infinite medium.

In this particular configuration of OPS, the source and the detector are physically overlapped and the incident beam is narrow, collimated, and normal to the wall of glass cuvette that holds the homogeneous medium. The diffusively backscattered energy flux is calculated by applying a time-dependent diffusion approach. In media with negligible absorption, the diffuse energy density $\Psi(\mathbf{r}, t)$ satisfies a diffusion equation:

$$\partial\Psi(\mathbf{r}, t)/\partial t = D\nabla^2\Psi(\mathbf{r}, t) + \delta(\mathbf{r})\delta(t) \quad (3)$$

where $\delta(\mathbf{r})\delta(t)$ is the impulse source at time $t = 0$ and $r = 0$ and D is the diffusion coefficient given by $D = (v \cdot l_t)/3$. Setting an appropriate boundary condition such that Ψ vanishes on the plane $z = -z_0$, the diffusion equation can be solved using the image source method (see, for instance, Ref.¹⁸) and the energy flux is obtained from Fick's law.¹⁹ Assuming an average energy transport velocity v , the pathlength dependence of the energy flux detected in the particular OPS geometry ($r = 0$) can be

evaluated to be:

$$J(s) = (4\pi l_t/3)^{-3/2} \cdot z_0 v \cdot s^{-5/2} \exp\left(-\frac{3z_0^2}{4sl_t}\right), \quad (4)$$

where the extrapolation length $z_0 = 2/3(1 + R_{eff})/(1 - R_{eff})l_t$ carries the information about the effective reflectivity R_{eff} at the boundaries. The extrapolation length ratio z_0/l_t is critical for describing light diffusion inside bounded media and, by studying the angular dependence of diffusely transmitted light, it has been accurately measured for various interfaces. For our particular case of air/glass/water interface, its value is 1.77. In the next chapter, a fuller discussion on the subject of the extrapolation length ratio will be presented, including our most recent results. It is worth mentioning the $s^{-5/2}$ behavior of the energy flux corresponding to diffusive waves with large optical pathlengths.

The pathlength-resolved backscattered intensities corresponding to water suspensions of polystyrene microspheres with particle diameter of $0.426\mu\text{m}$ and various volume fractions are shown in Fig. 5-a. To account for instrumentation effects, we use a two-fold fitting procedure: first, the tail of

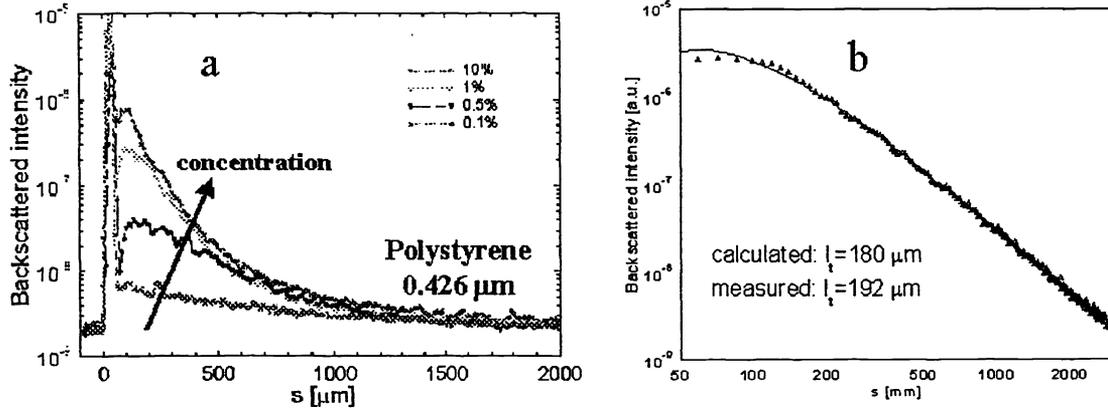


Figure 5: OPS experimental results for media of different concentrations (a) and an example of data fitted with Eq. 4 (b).

the distribution is fitted with $-5/2$ dependence to obtain an amplitude normalization constant and second, the whole distribution is fitted with the prediction of Eq. 4 to infer the value of l_t . The absorption lengths of the media used are roughly two orders of magnitude longer than the corresponding scattering lengths, supporting our assumption of negligible absorption. As we can see in Fig. 5-b, the diffusion theory, corrected for the effective reflectivity of the interface, makes a good description of the experimental data. Applying the Mie scattering theory, we also calculated the corresponding values of transport mean free paths for these media. Excellent agreement is obtained between the conventional estimations of l_t and the results obtained through our fitting procedure. This results suggest that reliable measurements of photon transport mean free path can be based on OPS.

The optical pathlength probability density $P(s)$ can be directly obtained by normalizing the low-coherence reflectivity data and, subsequently, one can infer characteristics such as the average optical pathlength as well as higher order moments for any kind of random medium. Detailed description of field penetration depth is important in many light delivery applications, such as LADAR techniques. For the sake of brevity, we neglected the absorption but incorporating its effects is straightforward.

We applied the principle of interference with partially coherent light to reconstruct the optical pathlength distribution of light scattered from a random medium with the purpose of background characterization. The results show that accurate data can be obtained about the transport mean free

path and that the less demanding instrumentation needed to record OPS data makes this approach appealing for a variety of applications, including remote sensing.

4 Conclusions

The low-coherence interferometry has been applied to retrieve information from an inhomogeneous distribution of scattering particles. Two configurations have been proposed in which both ballistic and diffusive light components were quantified. In the transmission measurements, the attenuation of waves due to scattering was evaluated, which allows obtaining the mean free path of the medium. Optical pathlength spectroscopy applies the same principle in a backscattering configuration and allows measuring the optical pathlength distribution of backscattered light and, therefore, the transport mean free path of the scattering medium. The two configurations presented complement each other, since the OPS applies to dense media that are not suitable for ballistic attenuation measurements. Investigating scattering media with low-coherence light is appealing for various applications due to the high dynamic range available and the high temporal resolution given by the short coherence length of the source.

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