## Diffraction Effects

The resolving power of a perfect optical system is determined by **diffraction** that results from the wave nature of light. An infinitely distant point source will image as a central peak surrounded by dark and faint bright rings, called the **Airy disk pattern**. Its intensity distribution and a theoretical image of a point source are shown here.

Relative Intensity

Relative Radius
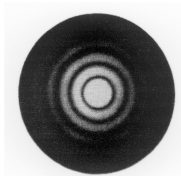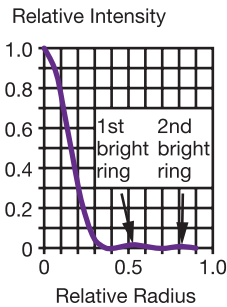
If the **Rayleigh criterion** is applied to resolution of double stars of equal luminance, their patterns overlap so that the peak of one lies at the first minimum of the other. The angular separation θ_R in arcseconds is then $\theta_R = 138/D_{EP}$, where D_{EP} is the instrument's entrance pupil diameter in mm. The empirical **Dawes criterion** for these double stars $\theta_D = 116/D_{EP}$ supports this theory. Either of these criteria can also be applied approximately to observed detail in terrestrial objects.

The minimum resolution of the eye is about one arcminute. The minimum magnification necessary to resolve images at the diffraction limit is dependent on D_{EP} and is ~0.43× per mm of aperture. This would be ~22× for a 50-mm objective. Higher magnification, such as 2× per mm, makes observing details easier for the eye. The excess of magnification beyond 0.43× per mm of aperture is called **empty magnification** and does not improve resolution.

Aberrations of the optical system reduce the height of the Airy disk central maximum. The **Strehl ratio** is the ratio of the actual height to the theoretical height. A ratio of 80% represents the **Marechal criterion**.

Obscuration Effects

Diffraction (see page 57) applies to unobscured apertures. Cassegrain and Newtonian scopes have circular central **obscurations**. They decrease **transmission** T, reduce the Strehl ratio S, and lower MTF in the images. We define obscuration ε as the ratio of the obscuration diameter D_{OBS} to the entrance pupil diameter D_{EP}. It typically ranges from 0.20 to 0.33, the values of which correspond to S of 0.92 (excellent image quality) and S of 0.80, (the Marechal criterion), with central obscuration: $T_{OBS} = 1 - \varepsilon^2$ and $S_{OBS} = (1 - \varepsilon^2)^2$.
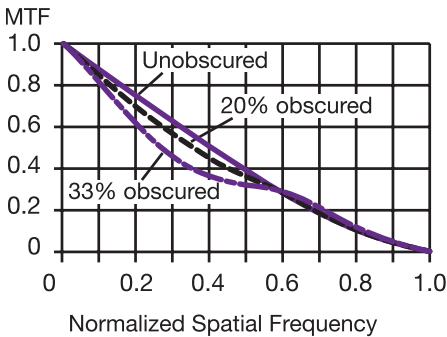
MTF is plotted here for three values of obscuration: $\varepsilon = 0$, (unobscured), $\varepsilon = 0.20$ (typical of a Newtonian scope), and $\varepsilon = 0.33$ (typical of a Schmidt– or a Maksutov–Cassegrain scope). The normalized spatial frequency is $\nu_n = \nu\lambda/D_{EP}$. Here ν and ν_n are in cycles/mm.

As a rule of thumb, MTF of an obscured system at low-to-mid spatial frequencies is equivalent to that of an unobscured system with aperture diameter equal to the diameter of the obscured system minus the diameter of the obscuration.

The spider supporting a secondary or folding flat mirror also obscures a small fraction of the aperture. Typically, one, three, or four vanes are symmetrically arranged about the axis to provide progressively stiffer support for the mirror. Each vane produces two **diffraction spikes** in star images. These spikes are always oriented perpendicular to the vane and result from diffraction by the straight edges. Curved vanes can reduce the spikes.

Atmospheric Scatter Effects

Light scatter by atmospheric haze reduces contrast and degrades resolution of distant objects seen vertically or horizontally through a scope or binocular. These effects depend on the **visual range** R_V, which is defined as the distance at which the unaided eye can detect an extended dark object having 2% contrast against the horizon sky. In clear daylight, R_V is >10 km. In light haze, it is 5 to 10 km and, in moderate haze, it is 2 to 5 km.

Ricco's Law states that the product of target angular size and its contrast is constant at the detection limit. Additional magnification compensates for contrast loss.

Detection of a target at distance L through a binocular or scope requires a minimum magnification of $M = (L/r)(e^x)$ where e is the Napierian logarithm base, r is the distance at which that same target can be detected with the unaided eye, and $x = 1.956(L - r)/R_V$.

L = Detection Distance (m)
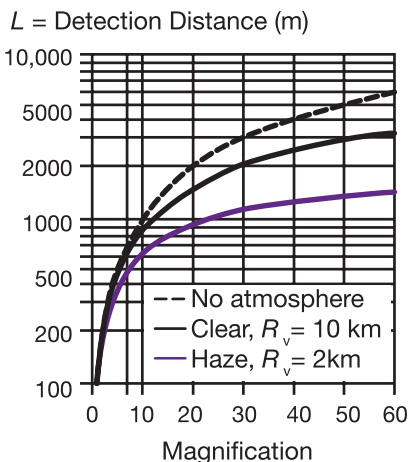
This graph shows the maximum values of L as functions of M and R_V for three atmospheric conditions when the same target can be detected at 100 m with the unaided eye. Note that a given change in M has less effect on L when the target is at a long distance.

For example, if R_V is 10 km (clear day) and r = 100 m, L = ~2500 m at 40× and ~2900 m at 50× (~16% change). For R_V of 2 km (moderate haze), L = ~1270 m at 40× and ~1400 m at 50× (~10% change).

Atmospheric Seeing Effects (Elevated Path)

Changes in the index of refraction of air between the observer and the target distort the transmitted wavefront and reduce image quality. This is called "**seeing**." There are three types of seeing: **scintillation** (variation in luminance), **image motion**, and **image blur**. All change rapidly with time. They are produced by local turbulence effects (due to temperature changes of the telescope and wind at the observation site), boundary layer effects in the atmosphere above the site (influenced by diurnal heating), and upper-atmospheric effects (from high-velocity winds at altitudes >10 km).

Because of seeing, the angular size of a star viewed through a perfect scope appears larger than it would from diffraction alone. Its size is then called the **seeing disc**.

The **Fried parameter** r_0 is the **coherence length** of the atmosphere. It is related to the size of the seeing disc d_S in radians at the zenith by the parameter $r_0 = 0.98\lambda/d_S$, where λ is the wavelength in mm. Scopes with apertures smaller than r_0 are diffraction and aberration limited. The Fried parameter is weakly dependent on wavelength (as $\lambda^{6/5}$) and on the angular distance z from the zenith [as $(\sec^2 z)^{-3/5}$]. During the day, the average value of r_0 is 20 to 40 mm. At night, the average value is ~100 mm. Values of $r_0 \approx 200$ mm are achieved at the best observatory sites.

The ratio D_{EP}/r_0 determines the effect of atmospheric seeing on the image. When $D_{EP}/r_0 < 3.7$, atmospheric seeing primarily produces image motion. For $D_{EP}/r_0 > 3.7$, the effect is image blur. In the latter case, the resolution is independent of telescope aperture size at ~$1.273\lambda/r_0$. Optimum resolution results at $D_{EP}/r_0 \approx 3$.

During the night, a well-adjusted amateur telescope with aperture smaller than $D_{EP} = 3r_0 \approx (3)(100 \text{ mm}) \approx 300$ mm may be able to provide diffraction-limited resolution for observing celestial objects.

Atmospheric Seeing (Horizontal Path)

When imaging along a horizontal path of length L, the Fried parameter r_0 varies with distance and is combined with another atmospheric turbulence parameter, the **index of refraction structure parameter** C_n^2 (expressed in units of $m^{-2/3}$), by the relationship $r_0 = 0.185(\lambda^2/LC_n^2)^{3/5}$. Typical values of C_n^2 range from $10^{-13}\,m^{-2/3}$ (medium effect) at the middle of the day to $10^{-14}\,m^{-2/3}$ (weak effect) at twilight.

Resolved Detail (mm)
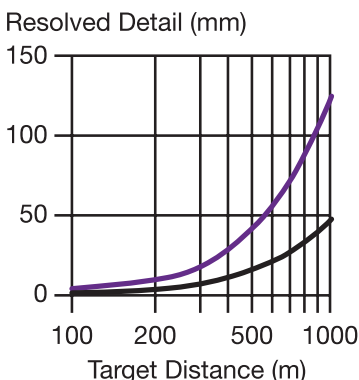
Target Distance (m)

This graph plots resolved detail (in mm) at the target distance L (in m) on a horizontal path for medium air turbulence (colored curve) and weak turbulence (black curve). As expected, increased turbulence reduces the amount of detail that can be resolved at any distance.

The **Greenwood Frequency** is correlated with the image motion in the focal plane of the viewing instrument produced by atmospheric seeing. It typically is 20 to 100 Hz. There is significant seeing-induced image motion of the image that reduces resolution during the ~50 msec integration time of the human eye.

> Atmospheric turbulence severely limits the resolution of spotting scopes and binoculars at long distances. This effect is independent of the optical quality of the instrument. Since r_0 is about 20 mm on horizontal paths, the largest effective instrument aperture for maximum resolution is $D_{EP} \approx 3r_0 \approx 60$ mm.